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The Rodrigues Operator Transform, Tables of Generalized Rodrigues Formulas

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THE RODRIGUES OPERATOR TRANSFORM,

TABLES OF GENERALIZED RODRIGUES FORMULAS

by

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ABSTRACT

Tables of generalized Rodrigues formulas for various special functions are given to facilitate use of the ideas in the author's "The Rodrigues Operator Transform, Preliminary Report," Boeing document D1-82-0492.

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INTRODUCTION

These tables give formulas for various special functions in terms of the Rodrigues operators. These operators are defined by

(1) $A(\alpha)t^{\mu} = \dfrac{\Gamma(\mu+\alpha+1)}{\Gamma(\mu+1)}\, t^{\mu} \qquad \mu+\alpha \neq -1,-2,\ldots$

(2) $\bar{A}(\alpha)t^{\mu} = \dfrac{\Gamma(\mu+1)}{\Gamma(\mu+\alpha+1)}\, t^{\mu} \qquad \mu \neq -1,-2,\ldots$

(3) $B(\alpha)t^{\mu} = \dfrac{\Gamma(\alpha-\mu)}{\Gamma(-\mu)}\, t^{\mu} \qquad \alpha-\mu \neq 0,-1,-2,\ldots$

(4) $\bar{B}(\alpha)t^{\mu} = \dfrac{\Gamma(-\mu)}{\Gamma(\alpha-\mu)}\, t^{\mu} \qquad \mu \neq 0,1,2,\ldots$

(5) $Mt^{\mu} = \Gamma(\mu+1)t^{\mu} \qquad \mu \neq -1,-2,\ldots$

(6) $\bar{M}t^{\mu} = \dfrac{1}{\Gamma(\mu+1)}\, t^{\mu}$

(7) $Nt^{\mu} = \Gamma(-\mu)t^{\mu} \qquad \mu \neq 0,1,2,\ldots$

(8) $\bar{N}t^{\mu} = \dfrac{1}{\Gamma(-\mu)}\, t^{\mu}.$

The first four are related to the operators of fractional integration and the second four to the Laplace transform and its inversion.

The details of the application of these formulations are given in "The Rodrigues Operator Transform, Preliminary Report," by T. P. Higgins.

These tables can be used to obtain many more relations. If we use

$$Rf(t) = t^{-1}f(\frac{1}{t}),$$

then, for example, from the formula

$$f(t) = A(\alpha)\tilde{A}(\beta)MA(\gamma)g(t),$$

it follows that

$$f(\frac{1}{t}) = B(\alpha+1)\tilde{B}(\beta+1)A(\gamma+1)Ng(\frac{1}{t}).$$

Many, but certainly not all, of the relations which can be obtained in this way have been included in the tables. Using the techniques given in the Preliminary Report, integral transform tables such as the Bateman Manuscript Project can be used to derive additional formulas.

The notation conforms to that of the Bateman series. The symbols are listed at the end of the tables.

<u>Elementary functions</u>

1. $(1+t)^{-\nu} = \frac{1}{\Gamma(\nu)} A(\nu-1)Me^{-t}$

2. $-\log(t+1) = A(-1)M(e^{-t}-1)$

3. $(1+t)^{-\nu} = \frac{1}{\Gamma(\nu)} Nt^{-\nu}e^{-1/t}$

4. $-\log(\frac{t}{t+1}) = N\{t(e^{-1/t}-1)\}$

5. $(1+t)^{\mu-\lambda} = \frac{\Gamma(\lambda)}{\Gamma(\lambda-\mu)} A(\lambda-\mu-1)\tilde{A}(\lambda-1)(t+1)^{-\lambda}$

6. $(1+t)^{\mu-\lambda} = \frac{\Gamma(\lambda)}{\Gamma(\lambda-\mu)} \tilde{B}(\mu)t^{\mu}(t+1)^{-\lambda}$

7. $.(t-1)^{\nu} = \Gamma(\nu+1)\tilde{M}t^{\nu}e^{-1/t}$

8. $H(t-1) = \tilde{M}e^{-1/t}$

9. $H(t-1) = \tilde{B}(1)\tilde{N}(1-e^{-t})$

10. $.(1-t)^{\nu} = \Gamma(\nu+1)\tilde{B}(\nu+1)\tilde{N}e^{-t}$

11. $H(1-t) = \tilde{B}(1)\tilde{N}e^{-t}$

12. $H(1-t) = \widetilde{M}[1-e^{-1/t}]$

13. $e^{t} = \frac{1}{\sqrt{\pi}} A(+\tfrac{1}{2}) M t^{\frac{1}{2}} \sinh(2\sqrt{t})$

14. $e^{-t} = \widetilde{M}(1+t)^{-1}$

15. $e^{-t} = \frac{1}{\Gamma(\nu+1)} B(\nu+1) N.(1-t)^{\nu}$

16. $e^{-t} = B(1) N H(1-t)$

17. $e^{-t} = -A(-1) t e^{-t} + 1$

18. $e^{-t} = \widetilde{B}(\mu) t^{\mu} e^{-t}$

19. $e^{-t} = \frac{1}{\sqrt{\pi}} B(\tfrac{3}{2}) N t^{-\frac{1}{2}} \sinh(2t^{-\frac{1}{2}})$

20. $e^{-1/t} = \widetilde{B}(1)\widetilde{N} t(1+t)^{-1}$

21. $e^{-1/t} = \frac{1}{\Gamma(\nu+1)} A(\nu) M t^{-\nu}.(t-1)^{\nu}$

22. $e^{-1/t} = M H(t-1)$

23. $e^{-t^{\frac{1}{2}}} = \frac{1}{\sqrt{\pi}} B(\tfrac{1}{2}) N e^{-t/4}$

24. $e^{-t^{\frac{1}{2}}} = \frac{1}{2\sqrt{\pi}} N t^{\frac{1}{2}} e^{-t/4}$

25. $e^{-1/t} = A(-1)\widetilde{A}(\mu-1) t^{-\mu} e^{-1/t}$

26. $e^{-t^{\frac{1}{2}}} = 2B(\frac{1}{2})\tilde{B}(-\frac{1}{2})t^{-\frac{1}{2}}e^{-t^{\frac{1}{2}}}$

27. $e^{-t^{-\frac{1}{2}}} = \frac{1}{\sqrt{\pi}} A(-\frac{1}{2})Me^{-1/4t}$

28. $e^{-t^{-\frac{1}{2}}} = \frac{1}{2\sqrt{\pi}} A(-1)Mt^{-\frac{1}{2}}e^{-1/t}$

29. $e^{-1/t} = A(-1)\tilde{A}(\mu-1)t^{-\mu}e^{-1/t}$

30. $e^{-t^{-\frac{1}{2}}} = 2A(-\frac{1}{2})\tilde{A}(-\frac{3}{2})t^{\frac{1}{2}}e^{-t^{-\frac{1}{2}}}$

31. $\sin t = \tilde{M}t(1+t^2)^{-1}$

32. $\sin t = A(-1)t \cos t$

33. $\sin(t + \frac{\mu\pi}{2}) = \tilde{B}(\mu)t^{\mu}\sin t$

34. $\cos t = \tilde{M}(1+t^2)^{-1}$

35. $\cos t = A(1)t^{-1}\sin t$

36. $\cos(t + \frac{\mu\pi}{2}) = \tilde{B}(\mu)t^{\mu}\cos t$

37. $\tan^{-1}(t) = A(-1)M \sin t$

38. $\tan^{-1}(\frac{1}{t}) = N \sin(\frac{1}{t})$

39. $\sin(t^{\frac{1}{2}}) = \frac{\sqrt{\pi}}{2} \widetilde{M} t^{\frac{1}{2}} e^{-t/4}$

40. $\cos(t^{\frac{1}{2}}) = \sqrt{\pi}\, \widetilde{A}(-\frac{1}{2}) \widetilde{M} e^{-t/4}$

41. $\cos(t^{\frac{1}{2}}) = 2A(\frac{1}{2}) \widetilde{A}(-\frac{1}{2}) t^{-\frac{1}{2}} \sin(t^{\frac{1}{2}})$

42. $\sin(t^{-\frac{1}{2}}) = \frac{\sqrt{\pi}}{2} \widetilde{B}(1) \widetilde{N} e^{-1/4t}$

43. $\cos(t^{-\frac{1}{2}}) = \sqrt{\pi}\, \widetilde{B}(\frac{1}{2}) \widetilde{N} e^{-1/4t}$

44. $\cos(t^{-\frac{1}{2}}) = 2B(\frac{3}{2}) \widetilde{B}(\frac{1}{2}) t^{\frac{1}{2}} \sin(t^{-\frac{1}{2}})$

45. $\sinh(t^{\frac{1}{2}}) = \frac{\sqrt{\pi}}{2} \widetilde{M} t^{\frac{1}{2}} e^{t/4}$

46. $\cosh(t^{\frac{1}{2}}) = \sqrt{\pi}\, \widetilde{A}(-\frac{1}{2}) \widetilde{M} e^{t/4}$

47. $\sinh(t^{-\frac{1}{2}}) = \frac{\sqrt{\pi}}{2} \widetilde{B}(1) \widetilde{N} t^{-\frac{1}{2}} e^{1/4t}$

48. $\cosh(t^{-\frac{1}{2}}) = \sqrt{\pi}\, \widetilde{B}(\frac{1}{2}) \widetilde{N} e^{t/4}$

Polynomials

1. $P_n^{\mu,-\mu}(1-t) = \frac{1}{\;}\quad\frac{n+1)}{!}\,\tilde{A}(\mu)P_n(1-t)$

2. $P_n^{(\lambda+\mu-\frac{1}{2},\lambda-\mu-\frac{1}{2})}(1-t) = \frac{\Gamma(\lambda+\mu+n+\frac{1}{2})\Gamma(2\lambda)}{\Gamma(\lambda+\frac{1}{2})\Gamma(2\lambda+n)}\,A(\lambda-\frac{1}{2})\tilde{A}(\lambda+\mu-\frac{1}{2})C_n^{\lambda}(1-t)$

3. $C_{2n}^{\lambda}(t^{\frac{1}{2}}) = \frac{\sqrt{\pi}}{n!\Gamma(\lambda)}\,A(n+\lambda-1)\tilde{A}(-\frac{1}{2})(t-1)^n$

4. $C_{2n+1}^{\lambda}(t^{\frac{1}{2}}) = \frac{\sqrt{\pi}}{n!\Gamma(\lambda)}\,A(n+\lambda-\frac{1}{2})t^{\frac{1}{2}}(t-1)^n$

5. $C_{2n}^{\lambda}(t^{-\frac{1}{2}}) = \frac{\sqrt{\pi}}{n!\Gamma(\lambda)}\,B(n+\lambda)\tilde{B}(\frac{1}{2})t^{-n}(1-t)^n$

6. $C_{2n+1}^{\lambda}(t^{-\frac{1}{2}}) = \frac{\sqrt{\pi}}{n!\Gamma(\lambda)}\,B(n+\lambda+\frac{1}{2})\tilde{B}(1)t^{-\frac{1}{2}-n}(1-t)^n$

7. $.(1-t)^{\lambda-\frac{1}{2}}C_{2n}^{\lambda}(t^{\frac{1}{2}}) = \frac{2^{2n}\Gamma(\lambda+n+\frac{1}{2})}{\Gamma(\lambda)\Gamma(2n+1)}\,\tilde{B}(\frac{1}{2}-n)t^{\frac{1}{2}}.(1-t)^{\lambda+n-1}$

8. $.(1-t)^{\lambda-\frac{1}{2}}C_{2n+1}^{\lambda}(t^{\frac{1}{2}}) = \frac{2^{2n+1}\Gamma(\lambda+n+\frac{1}{2})}{\Gamma(\lambda)\Gamma(2n+2)}\,B(\frac{1}{2})\tilde{B}(-n).(1-t)^{\lambda+n}$

9. $.(t-1)^{\lambda-\frac{1}{2}}C_{2n}^{\lambda}(t^{-\frac{1}{2}}) = \frac{2^{2n}\Gamma(\lambda+n+\frac{1}{2})}{\Gamma(\lambda)\Gamma(2n+1)}\,A(-\lambda-\frac{1}{2})\tilde{A}(-n-\lambda)t^{-n}.(t-1)^{\lambda+n-1}$

10. $.(t-1)^{\lambda-\frac{1}{2}}C_{2n+1}^{\lambda}(t^{-\frac{1}{2}}) = \frac{2^{2n+1}\Gamma(\lambda+n+\frac{1}{2})}{\Gamma(\lambda)\Gamma(2n+2)}\,A(-\lambda)\tilde{A}(-n-\lambda-\frac{1}{2})t^{-n-\frac{1}{2}}.(t-1)^{\lambda+n}$

11. $T_{2n}(t^{\frac{1}{2}}) = \frac{\sqrt{\pi}}{\Gamma(n)}\,A(n-1)\tilde{A}(-\frac{1}{2})(t-1)^n$

12. $T_{2n+1}(t^{\frac{1}{2}}) = \frac{\sqrt{\pi}\;(n+\frac{1}{2})}{n!} A(n - \frac{1}{2})t^{\frac{1}{2}}(t-1)^n$

13. $T_{2n}(t^{-\frac{1}{2}}) = \frac{\sqrt{\pi}}{\Gamma(n)} B(n)\bar{B}(\frac{1}{2})t^{-n}(1-t)^n$

14. $T_{2n+1}(t^{-\frac{1}{2}}) = \frac{\sqrt{\pi}\;(n+\frac{1}{2})}{n!} B(n + \frac{1}{2})\bar{B}(1)t^{-n-\frac{1}{2}}(1-t)^n$

15. $H(1-t)(1-t)^{-\frac{1}{2}}T_{2n}(t^{\frac{1}{2}}) = \frac{\sqrt{\pi}}{\Gamma(n)} \bar{B}(\frac{1}{2} - n)t^{\frac{1}{2}}.(1-t)^{n-1}$

16. $H(1-t)(1-t)^{-\frac{1}{2}}T_{2n+1}(t^{\frac{1}{2}}) = \frac{\sqrt{\pi}}{n!} B(\frac{1}{2})\bar{B}(-n).(1-t)^n$

17. $H(t-1)(t-1)^{-\frac{1}{2}}T_{2n}(t^{-\frac{1}{2}}) = \frac{\sqrt{\pi}}{\Gamma(n)} A(-\frac{1}{2})\bar{A}(-n)t^{-n}.(t-1)^{n-1}$

18. $H(t-1)(t-1)^{-\frac{1}{2}}T_{2n+1}(t^{-\frac{1}{2}}) = \frac{\sqrt{\pi}}{n!} \bar{A}(-n - \frac{1}{2})t^{-\frac{1}{2}-n}.(t-1)^n$

19. $P_{2n}(t^{\frac{1}{2}}) = \frac{1}{n!} A(n - \frac{1}{2})\bar{A}(- \frac{1}{2})(t-1)^n$

20. $P_{2n+1}(t^{\frac{1}{2}}) = \frac{1}{n!} A(n)t^{\frac{1}{2}}(t-1)^n$

21. $H(1-t)P_{2n}(t^{\frac{1}{2}}) = \frac{1}{\Gamma(n+\frac{1}{2})} \bar{B}(\frac{1}{2} - n)t^{\frac{1}{2}}.(1-t)^{n-\frac{1}{2}}$

22. $H(1-t)P_{2n+1}(t^{\frac{1}{2}}) = \frac{1}{\Gamma(n + \frac{3}{2})} B(\frac{1}{2})\bar{B}(-n).(1-t)^{n+\frac{1}{2}}$

23. $P_{2n}(t^{-\frac{1}{2}}) = \frac{1}{n!} B(n + \frac{1}{2})\bar{B}(\frac{1}{2})t^{-n}(t-1)^n$

24. $P_{2n+1}(t^{-\frac{1}{2}}) = \frac{1}{n!} B(n+1)\bar{B}(1)t^{-n-\frac{1}{2}}(t-1)^{n}$

25. $H(t-1)P_{2n}(t^{-\frac{1}{2}}) = \frac{1}{\Gamma(n+\frac{1}{2})} A(-1)\bar{A}(-n-\tfrac{1}{2})t^{-n}.(t-1)^{n-\frac{1}{2}}$

26. $H(t-1)P_{2n+1}(t^{-\frac{1}{2}}) = \frac{1}{\Gamma(n + \frac{3}{2})} A(-\tfrac{1}{2})\bar{A}(-n-1)t^{-n-\frac{1}{2}}.(t-1)^{n+\frac{1}{2}}$

27. $L_n^{\alpha}(t) = \frac{\Gamma(\alpha+n+1)}{n!} \bar{A}(\alpha)\bar{M}(1-t)^{n}$

28. $L_n^{\alpha}(t^{-1}) = \frac{\Gamma(\alpha+n+1)}{n!} \bar{B}(\alpha+1)\bar{N}t^{-n}(t-1)^{n}$

29. $L_n^{\alpha}(t) = \frac{1}{n!} e^{t}A(\alpha+n)\bar{A}(\alpha)e^{-t}$

30. $.(t-1)^{\alpha}L_n^{\alpha}(t-1) = \frac{e^{-t}}{n!} A(n)t^{-n}e^{t}.(t-1)^{\alpha+n}$

31. $L_n^{\alpha}(t^{-1}) = \frac{1}{n!} e^{1/t}B(\alpha+n+1)\bar{B}(\alpha+1)e^{-1/t}$

32. $L_n^{\alpha}(t) = \frac{\Gamma(\alpha+n+1)\Gamma(\lambda)}{\Gamma(n+1)\Gamma(\alpha+1)\Gamma(\lambda+\mu)} A(\mu+\lambda-1)\bar{A}(\lambda-1)\,{}_2F_2(-n,\lambda;\alpha+1,\lambda+\mu;t)$

33. $L_n^{\alpha}(t^{-1}) = \frac{\Gamma(\alpha+n+1)\Gamma(\lambda)}{\Gamma(n+1)\Gamma(\alpha+1)\Gamma(\lambda+\mu)} B(\mu+\lambda)\bar{B}(\lambda)\,{}_2F_2(-n,\lambda;\alpha+1,\lambda+\mu;t^{-1})$

34. $L_n^{\alpha+\mu}(t) = \frac{\Gamma(\alpha+\mu+n+1)}{\Gamma(\alpha+n+1)} A(\alpha)\bar{A}(\mu+\alpha)L_n^{\alpha}(t)$

35. $L_n^{\alpha+\mu}(t^{-1}) = \frac{\Gamma(\alpha+\mu+n+1)}{\Gamma(\alpha+n+1)} B(\alpha+1)\bar{B}(\mu+\alpha)L_n^{\alpha}(t^{-1})$

36. $$(1\mp t)^{\beta+\mu}P_n^{(\alpha,\beta+n)}(1\mp 2t) = \frac{\Gamma(-\beta-\mu)}{\Gamma(-\beta-\mu-n)} A(-\beta-\mu-n-1)\tilde{A}(-\beta-n-1) \cdot$$

$$\cdot\ (1-t)^{\beta}P_n^{(\alpha,\beta)}(1\mp 2t)$$

37. $$P_n^{(\alpha,\beta)}(1\mp 2t) = \frac{\Gamma(\alpha+n+1)\Gamma(\lambda)}{n!\,\Gamma(\alpha+1)\Gamma(\lambda+\mu)} A(\lambda+\mu-1)\tilde{A}(\lambda-1) \cdot$$

$$\cdot\ {}_3F_2(-n,n+\alpha+\beta+1,\lambda;\alpha+1,\lambda+\mu;\pm t)$$

38. $$P_n^{(\alpha,\beta)}(2t\mp 1) = \frac{(-1)^n\Gamma(\beta+n+1)\Gamma(\lambda)}{n!\,\Gamma(\beta+1)\Gamma(\lambda+\mu)} A(\lambda+\mu-1)\tilde{A}(\lambda-1) \cdot$$

$$\cdot\ {}_3F_2(-n,n+\alpha+\beta+1,\lambda;\beta+1,\lambda+\mu;\pm t)$$

39. $$(1\mp t)^{\beta}P_n^{(\alpha,\beta)}(1\mp 2t) = \frac{\Gamma(\alpha+n+1)\Gamma(\lambda)}{\Gamma(\alpha+1)\Gamma(\lambda+\mu)}\ {}_3F_2(\alpha+n+1,-\beta-n,\lambda;\alpha+1,\lambda+\mu;\pm t)$$

40. $$(1+t)^{\beta}P_n^{(\alpha,\beta)}(1+2t) = \frac{1}{n!\,\Gamma(-\beta-n)} A(\alpha+n)\tilde{A}(\alpha)A(-\beta-n-1)Me^{-t}$$

41. $$C_{2n}^{\lambda}(t^{\frac{1}{2}}) = \frac{(\lambda)_n\Gamma(\nu)(-1)^n}{n!\,\Gamma(\mu+\nu)} A(\nu+\mu-1)\tilde{A}(\nu-1)\,{}_3F_2(-n,n+\lambda,\nu;\tfrac{1}{2},\mu+\nu;t)$$

42. $$C_{2n+1}^{\lambda}(t^{\frac{1}{2}}) = \frac{2(\lambda)_{n+1}\Gamma(\nu+\frac{1}{2})(-1)^n}{n!\,\Gamma(\mu+\nu+\frac{1}{2})} A(\nu+\mu-1)\tilde{A}(\nu-1)t^{\frac{1}{2}} \cdot$$

$$\cdot\ {}_3F_2(-n,n+\lambda+1,\nu+\tfrac{1}{2};\tfrac{3}{2},\mu+\nu+\tfrac{1}{2};t)$$

43. $$P_n^{\alpha+\mu,\beta-\mu}(1-t) = \frac{\Gamma(\alpha+\mu+n+1)}{\Gamma(\alpha+n+1)} A(\alpha)\tilde{A}(\mu+\alpha)P_n^{\alpha,\beta}(1-t)$$

44. $$P_n^{\alpha-\mu,\beta+\mu}(t-1) = \frac{\Gamma(\beta+\mu+n+1)}{\Gamma(\beta+n+1)} A(\beta)\widetilde{A}(\beta+\mu)P_n^{\alpha,\beta}(t-1)$$

<u>Legendre functions</u>

1. $$\cdot(1-t)^{-\mu/2}P_\nu^\mu(t^{\frac{1}{2}}) = \frac{2^\mu}{\Gamma(\frac{1+\nu-\mu}{2})}\widetilde{B}\left(\frac{1-\nu-\mu}{2}\right)t^{\frac{1}{2}}.(1-t)^{(\nu-\mu-1)/2}$$

2. $$\cdot(1-t)^{-\mu/2}P_\nu^\mu(t^{\frac{1}{2}}) = \frac{2^\mu}{\Gamma(-\frac{\nu+\mu}{2})}\widetilde{B}\left(1+\frac{\nu-\mu}{2}\right)t^{\frac{1}{2}}.(1-t)^{-1-(\nu+\mu)/2}$$

3. $$\cdot(1-t)^{-\mu/2}P_\nu^\mu(t^{\frac{1}{2}}) = 2^\mu\widetilde{B}\left(\frac{1-\mu-\nu}{2}\right)\widetilde{B}\left(1-\frac{\mu-\nu}{2}\right)\widetilde{N}t^{\frac{1}{2}}e^{-t}$$

4. $$\cdot(1-t)^{-\mu/2}P_\nu^\mu(t^{\frac{1}{2}}) = 2^\mu B\left(\frac{1}{2}\right)\widetilde{B}\left(\frac{1-\mu-\nu}{2}\right)\widetilde{B}\left(1-\frac{\mu-\nu}{2}\right)\widetilde{N}e^{-t}$$

5. $$\cdot(t-1)^{-\mu/2}P_\nu^\mu(t^{-\frac{1}{2}}) = \frac{2^\mu}{\Gamma(\frac{1+\nu-\mu}{2})}A\left(\frac{\mu}{2}-1\right)\widetilde{A}\left(-\frac{\nu+1}{2}\right)t^{-\nu/2}.(t-1)^{(\nu-\mu-1)/2}$$

6. $$\cdot(t-1)^{-\mu/2}P_\nu^\mu(t^{-\frac{1}{2}}) = \frac{2^\mu}{\Gamma(-\frac{\nu+\mu}{2})}A\left(\frac{\mu}{2}-1\right)\widetilde{A}\left(\frac{\nu}{2}\right)t^{(\nu-1)/2}.(t-1)^{-1-(\nu+\mu)/2}$$

7. $$\cdot(t-1)^{-\mu/2}P_\nu^\mu(t^{-\frac{1}{2}}) = 2^\mu A\left(\frac{\mu}{2}-1\right)\widetilde{A}\left(-\frac{\nu+1}{2}\right)\widetilde{A}\left(\frac{\nu}{2}\right)\widetilde{M}t^{-(\mu+1)/2}e^{-1/t}$$

8. $$\cdot(t-1)^{-\mu/2}P_\nu^\mu(t^{-\frac{1}{2}}) = 2^\mu A\left(\frac{\mu-1}{2}\right)\widetilde{A}\left(-\frac{\nu+1}{2}\right)\widetilde{A}\left(\frac{\nu}{2}\right)\widetilde{M}t^{-\mu/2}e^{-1/t}$$

9. $$P_\nu^{\lambda-\mu}(1+2t) = (1+t)^{(\lambda-\mu)/2}A\left(-\frac{\mu+\lambda}{2}\right)\widetilde{A}\left(\frac{\mu-\lambda}{2}\right)t^{\mu/2}(1+t)^{-\lambda/2}P_\nu^\lambda(1+2t)$$

10. $$\cdot(1-t)^{(\mu-\lambda)/2}P_\nu^{\lambda-\mu}(1-2t) = A\left(-\frac{\mu+\lambda}{2}\right)\widetilde{A}\left(\frac{\mu-\lambda}{2}\right)t^{\mu/2}.(1-t)^{-\lambda/2}P_\nu^\lambda(1-2t)$$

11. $$\cdot(1-t)^{-\mu/2}P_\nu^\mu(t^{\frac{1}{2}}) = \sqrt{\pi}\,2^\mu\widetilde{B}\left(\frac{1-\mu-\nu}{2}\right)\widetilde{B}\left(1-\frac{\mu-\nu}{2}\right)\widetilde{NNB}(1)t^{\frac{1}{2}}e^{-2t^{\frac{1}{2}}}$$

12. $\cdot(1-t)^{-\mu/2}P_\nu^\mu(t^{\frac{1}{2}}) = \sqrt{\pi}\ 2^\mu\widetilde{B}\left(\frac{1-\mu-\nu}{2}\right)\widetilde{B}\left(1 - \frac{\mu-\nu}{2}\right)\widetilde{NN}e^{-2t^{\frac{1}{2}}}$

13. $\cdot(t-1)^{-\mu/2}P_\nu^\mu(t^{-\frac{1}{2}}) = \sqrt{\pi}\ 2^\mu A\left(\frac{\mu}{2} - 1\right)\widetilde{A}\left(-\frac{\nu+1}{2}\right)\widetilde{A}\left(\frac{\nu}{2}\right)\widetilde{A}\left(\frac{\nu}{2}\right)\widetilde{A}\left(\frac{\mu}{2}\right)\widetilde{MM}t^{(-\mu+1)/2}e^{-2t^{-\frac{1}{2}}}$

14. $\cdot(t-1)^{-\mu/2}P_\nu^\mu(t^{-\frac{1}{2}}) = \sqrt{\pi}\ 2^\mu A\left(-\frac{\nu+1}{2}\right)A\left(\frac{\nu}{2}\right)MMt^{-\mu/2}e^{-2t^{-\frac{1}{2}}}$

15. $\cdot(t^2-1)^{\mu/2}Q_\nu^{-\mu}(t) = e^{-i\pi\mu}A(-\mu)t^{-\mu}Q_\nu(t)$

16. $\cdot(t^2-1)^{(\lambda+\mu)/2}Q_\nu^{-\mu-\lambda}(t) = e^{-i\pi\mu}A(-\mu)t^{-\mu}Q_\nu^{-\lambda}(t)$

Bessel functions

1. $e^{it}J_\nu(t) = \frac{2^\nu}{\sqrt{\pi}} A(-\frac{1}{2})\tilde{A}(\nu)t^\nu e^{2it}$

2. $\sin t\, J_\nu(t) = \frac{2^\nu}{\sqrt{\pi}} A(-\frac{1}{2})\tilde{A}(\nu)t^\nu \sin(2t)$

3. $\sin(t^{-1})J_\nu(t^{-1}) = \frac{2^\nu}{\sqrt{\pi}} B(\frac{1}{2})\tilde{B}(\nu+1)t^{-\nu}\sin(\frac{2}{t})$

4. $\sin t\, J_\nu(t) = -\frac{2^{\nu-1}}{\sqrt{\pi}} A(-\frac{1}{2})\tilde{A}(\nu)A(1-\nu)\tilde{A}(-\nu)t^{\nu-1}\cos(2t)$

5. $\sin(t^{-1})J_\nu(t^{-1}) = -\frac{2^{\nu-1}}{\sqrt{\pi}} B(\frac{1}{2})\tilde{B}(\nu+1)B(2-\nu)\tilde{B}(1-\nu)t^{1-\nu}\cos(\frac{2}{t})$

6. $\cos t\, J_\nu(t) = \frac{2^\nu}{\sqrt{\pi}} A(-\frac{1}{2})\tilde{A}(\nu)t^\nu \cos(2t)$

7. $\cos t\, J_\nu(t) = \frac{2^{\nu-1}}{\sqrt{\pi}} A(-\frac{1}{2})\tilde{A}(\nu)A(1-\nu)\tilde{A}(-\nu)t^{\nu-1}\sin(2t)$

8. $\cos(t^{-1})J_\nu(t^{-1}) = \frac{2^\nu}{\sqrt{\pi}} B(\frac{1}{2})\tilde{B}(\nu+1)t^{-\nu}\cos(\frac{2}{t})$

9. $\cos(t^{-1})J_\nu(t^{-1}) = \frac{2^{\nu-1}}{\sqrt{\pi}} B(\frac{1}{2})\tilde{B}(\nu+1)B(2-\nu)\tilde{B}(1-\nu)t^{1-\nu}\sin(\frac{2}{t})$

10. $J_\nu(t^{\frac{1}{2}}) = \frac{2^{1-\nu}}{\sqrt{\pi}} A(\frac{1-\nu}{2})\tilde{A}(\frac{\nu}{2})t^{(\nu-1)/2}\sin(t^{\frac{1}{2}})$

11. $J_\nu(t^{\frac{1}{2}}) = \frac{2^\nu}{\sqrt{\pi}} B(\frac{\nu}{2})\tilde{B}(\frac{1-\nu}{2})t^{-\nu/2}\sin(t^{\frac{1}{2}})$

12. $$J_\nu(t^{\frac{1}{2}}) = \frac{2^{-\nu}}{\sqrt{\pi}} A\left(-\frac{\nu+1}{2}\right)\widetilde{A}\left(\frac{\nu}{2}\right)t^{\nu/2}\cos(t^{\frac{1}{2}})$$

13. $$J_\nu(t^{\frac{1}{2}}) = \frac{2^{\nu+1}}{\sqrt{\pi}} B\left(\frac{\nu}{2}\right)\widetilde{B}\left(-\frac{\nu+1}{2}\right)t^{-(\nu+1)/2}\cos(t^{\frac{1}{2}})$$

14. $$J_\nu(t^{\frac{1}{2}}) = \frac{2^{\nu}}{\sqrt{\pi}} B\left(\frac{\nu}{2}\right)\widetilde{B}\left(\frac{1-\nu}{2}\right)t^{-\nu/2}\sin(t^{\frac{1}{2}})$$

15. $$J_\nu(t^{-\frac{1}{2}}) = \frac{2^{1-\nu}}{\sqrt{\pi}} B\left(\frac{3-\nu}{2}\right)\widetilde{B}\left(\frac{\nu+2}{2}\right)t^{(1-\nu)/2}\sin(t^{-\frac{1}{2}})$$

16. $$J_\nu(t^{-\frac{1}{2}}) = \frac{2^{\nu}}{\sqrt{\pi}} A\left(\frac{\nu}{2}-1\right)\widetilde{A}\left(-\frac{\nu+1}{2}\right)t^{+\nu/2}\sin(t^{-\frac{1}{2}})$$

17. $$J_\nu(t^{-\frac{1}{2}}) = \frac{2^{\nu+1}}{\sqrt{\pi}} A\left(\frac{\nu}{2}-1\right)\widetilde{A}\left(-\frac{\nu+3}{2}\right)t^{(\nu+1)/2}\cos(t^{-\frac{1}{2}})$$

18. $$J_\nu(t^{-\frac{1}{2}}) = \frac{2^{-\nu}}{\sqrt{\pi}} B\left(\frac{1-\nu}{2}\right)\widetilde{B}\left(\frac{\nu+2}{2}\right)t^{-\nu/2}\cos(t^{-\frac{1}{2}})$$

19. $$J_\nu(t^{-\frac{1}{2}}) = \frac{2^{\nu}}{\sqrt{\pi}} A\left(\frac{\nu}{2}-1\right)\widetilde{A}\left(-\frac{\nu+1}{2}\right)t^{\nu/2}\sin(t^{-\frac{1}{2}})$$

20. $$J_\nu(t^{\frac{1}{2}}) = 2^{-\nu}\widetilde{A}\left(\frac{\nu}{2}\right)\widetilde{M}t^{\nu/2}e^{-t/4}$$

21. $$J_\nu(t^{-\frac{1}{2}}) = 2^{-\nu}\widetilde{B}\left(\frac{\nu}{2}+1\right)\widetilde{N}t^{-\nu/2}e^{-1/4t}$$

22. $$J_{\mu+\nu}(t^{\frac{1}{2}}) = 2^{-\mu}A\left(\frac{\nu-\mu}{2}\right)\widetilde{A}\left(\frac{\mu+\nu}{2}\right)t^{\mu/2}J_\nu(t^{\frac{1}{2}})$$

23. $$J_{\nu-\mu}(t^{\frac{1}{2}}) = 2^{-\mu}B\left(\frac{\nu-\mu}{2}\right)\widetilde{B}\left(\frac{\nu+\mu}{2}\right)t^{\mu/2}J_\nu(t^{\frac{1}{2}})$$

24. $J_{\mu+\nu}(t^{-\frac{1}{2}}) = 2^{-\mu} B(\frac{\nu-\mu+2}{2}) \tilde{B}(\frac{\mu+\nu+2}{2}) t^{-\mu/2} J_{\nu}(t^{-\frac{1}{2}})$

25. $J_{\nu-\mu}(t^{-\frac{1}{2}}) = 2^{-\mu} A(\frac{\nu-\mu-2}{2}) \tilde{A}(\frac{\mu+\nu-2}{2}) t^{-\mu/2} J_{\nu}(t^{-\frac{1}{2}})$

26. $J_{\nu}(t) = \frac{2^{-\nu}}{\Gamma(\lambda)\Gamma(\nu+1)} A(\lambda-\nu-1) M t^{\nu} {}_0F_3(\nu+1, \frac{1}{2}\lambda, \frac{\lambda+1}{2}; -(\frac{t}{4})^2)$

27. $J_{\nu}(t^{-1}) = \frac{2^{-\nu}}{\Gamma(\lambda)\Gamma(\nu+1)} B(\lambda-\nu) N t^{-\nu} {}_0F_3(\nu+1, \frac{1}{2}\lambda; \frac{\lambda+1}{2}; -(\frac{1}{4t})^2)$

28. $J_{\nu}(t^{\frac{1}{2}}) = \frac{2^{-\nu}}{\Gamma(\mu+\frac{1}{2}\nu)\Gamma(\nu+1)} A(\mu-1) M t^{\frac{1}{2}\nu} {}_0F_2(\mu + \frac{1}{2}\nu, \nu+1; -\frac{t}{4})$

29. $J_{\nu}(t^{-\frac{1}{2}}) = \frac{2^{-\nu}}{\Gamma(\mu+\frac{1}{2})\Gamma(\nu+1)} B(\mu) N t^{-\nu/2} {}_0F_2(\mu + \frac{\nu}{2}, \nu+1; -\frac{1}{4t})$

30. $[J_{\nu}(t^{\frac{1}{2}})]^2 = \frac{1}{\sqrt{\pi}} A(-\frac{1}{2}) \tilde{A}(\nu) t^{\nu/2} J_{\nu}(2t^{\frac{1}{2}})$

31. $[J_{\nu}(t^{\frac{1}{2}})]^2 = \frac{1}{\sqrt{\pi}} A(-\frac{1}{2}) \tilde{A}(\nu) \tilde{M} t^{\nu} e^{-t}$

32. $[J_{\nu}(t^{-\frac{1}{2}})]^2 = \frac{1}{\sqrt{\pi}} B(\frac{1}{2}) \tilde{B}(\nu+1) t^{-\nu/2} J_{\nu}(2t^{-\frac{1}{2}})$

33. $[J_{\nu}(t^{-\frac{1}{2}})]^2 = \frac{1}{\sqrt{\pi}} B(\frac{1}{2}) \tilde{B}(\nu+1) \tilde{B}(1) \tilde{N} t^{-\nu} e^{-1/t}$

34. $J_{\nu}(t^{\frac{1}{2}}) J_{\nu-1}(t^{\frac{1}{2}}) = \frac{1}{\sqrt{\pi}} \tilde{A}(\nu - \frac{1}{2}) t^{(\nu-1)/2} J_{\nu}(2t^{\frac{1}{2}})$

35. $J_{\nu}(t^{\frac{1}{2}}) J_{\nu-1}(t^{\frac{1}{2}}) = \frac{1}{\sqrt{\pi}} \tilde{A}(\nu - \frac{1}{2}) \tilde{A}(\frac{1}{2}) \tilde{M} t^{\nu-\frac{1}{2}} e^{-t}$

36. $J_\nu(t^{-\frac{1}{2}})J_{\nu-1}(t^{-\frac{1}{2}}) = \frac{B(1)}{\sqrt{\pi}}\,\tilde{B}(\nu+\frac{1}{2})t^{(1-\nu)/2}J_\nu(2t^{-\frac{1}{2}})$

37. $J_\nu(t^{-\frac{1}{2}})J_{\nu-1}(t^{-\frac{1}{2}}) = \frac{1}{\sqrt{\ }}\,B(1)\tilde{B}(\nu+\frac{1}{2})\tilde{B}(\frac{3}{2})\tilde{N}t^{\frac{1}{2}-\nu}e^{-1/t}$

38. $J_\nu(t^{\frac{1}{2}})Y_\nu(t^{\frac{1}{2}}) = \frac{1}{\sqrt{\pi}}\,A(-\frac{1}{2})\tilde{A}(\nu)t^{\nu/2}Y_\nu(2t^{\frac{1}{2}})$

39. $J_\nu(t^{-\frac{1}{2}})Y_\nu(t^{-\frac{1}{2}}) = \frac{1}{\sqrt{\pi}}\,B(\frac{1}{2})\tilde{B}(\nu+1)t^{-\nu/2}Y_\nu(2t^{-\frac{1}{2}})$

40. $J_\nu(t) = MJ_\nu(\sqrt{2}\ t^{\frac{1}{2}})I_\nu(\sqrt{2}\ t^{\frac{1}{2}})$

41. $J_\nu(\frac{1}{t}) = B(1)NJ_\nu(\sqrt{2}\ /t^{\frac{1}{2}})I_\nu(\sqrt{2}\ /t^{\frac{1}{2}})$

42. $J_\nu(2\alpha\beta t) = e^{(\beta^2-\alpha^2)t}MJ_\nu(2\beta t^{\frac{1}{2}})I_\nu(2\alpha t^{\frac{1}{2}})$

43. $J_\nu(2\alpha\beta t^{-1}) = e^{(\beta^2-\alpha^2)/t}B(1)NJ_\nu(2\beta t^{-\frac{1}{2}})I_\nu(2\alpha t^{-\frac{1}{2}})$

44. $[J_\nu(t^{\frac{1}{2}})]^2 = \frac{2^{-2\nu}\Gamma(\lambda)}{[\Gamma(\nu+1)]^2\Gamma(\lambda+\mu)}\,A(\lambda+\mu-\nu-1)\tilde{A}(\lambda-\nu-1)\,{}_2F_3(\lambda,\nu+\frac{1}{2};\lambda+\mu,\nu+1,2\nu+1;-t)$

45. $J_\nu(t^{\frac{1}{2}})J_{-\nu}(t^{\frac{1}{2}}) = \frac{\Gamma(\lambda)\sin(\pi\nu)}{(\pi\nu)\Gamma(\lambda+\mu)}\,A(\mu+\lambda-1)\tilde{A}(\lambda-1)\,{}_2F_3(\frac{1}{2},\lambda;1+\nu,1-\nu,\lambda+\mu;-t)$

46. $Y_\nu(t^{\frac{1}{2}}) = -\frac{2^\nu}{\sqrt{\pi}}\,B(\frac{\nu}{2})\tilde{B}(\frac{1-\nu}{2})t^{-\nu/2}\cos(t^{\frac{1}{2}})$

47. $Y_\nu(t^{\frac{1}{2}}) = \frac{2^{\nu+1}}{\sqrt{\pi}}\,B(\frac{\nu+1}{2})\tilde{B}(-\frac{\nu}{2})t^{-\nu/2}\sin(t^{\frac{1}{2}})$

48. $Y_\nu(t^{-\frac{1}{2}}) = -\frac{2^{-\nu}}{\sqrt{\pi}} A(\frac{\nu}{2} - 1)\tilde{A}(-\frac{\nu+1}{2})t^{\nu/2}\cos(t^{-\frac{1}{2}})$

49. $Y_\nu(t^{-\frac{1}{2}}) = \frac{2^{\nu+1}}{\sqrt{\pi}} A(\frac{\nu-1}{2})\tilde{A}(-\frac{\nu}{2} - 1)t^{\nu/2}\sin(t^{-\frac{1}{2}})$

50. $Y_{\nu-\mu}(t^{\frac{1}{2}}) = 2^{-\mu}B(\frac{\nu-\mu}{2})\tilde{B}(\frac{\nu+\mu}{2})t^{\mu/2}Y_\nu(t^{\frac{1}{2}})$

51. $Y_{\nu-\mu}(t^{-\frac{1}{2}}) = 2^{-\mu}A(\frac{\nu-\mu-2}{2})\tilde{A}(\frac{\nu+\mu-2}{2})t^{-\mu/2}Y_\nu(t^{-\frac{1}{2}})$

52. $\operatorname{ctn}(\pi\nu)[J_{\nu/2}(t^{\frac{1}{2}})]^2 - \csc(\pi\nu)[J_{-\nu/2}(t^{\frac{1}{2}})]^2 = \frac{1}{\sqrt{\pi}} A(-\frac{1}{2})Y_\nu(2t^{\frac{1}{2}})$

53. $[\cos(t)J_{\frac{1}{2}-\mu}(t)-\sin(t)Y_{\frac{1}{2}-\mu}(t)] = \frac{2^{\mu+\frac{1}{2}}}{\sqrt{\pi}} B(\frac{1}{2} - \mu)\tilde{B}(\frac{1}{2})t^{\mu-\frac{1}{2}}\sin t$

54. $[\sin(t)J_{\frac{1}{2}-\mu}(t)+\cos(t)Y_{\frac{1}{2}-\mu}(t)] = \frac{2^{\mu+\frac{1}{2}}}{\sqrt{\pi}} B(\frac{1}{2} - \mu)\tilde{B}(\frac{1}{2})t^{\mu-\frac{1}{2}}\cos t$

55. $[J_\nu(t^{\frac{1}{2}})Y_{-\nu}(t^{\frac{1}{2}}) + J_{-\nu}(t^{\frac{1}{2}})Y_\nu(t^{\frac{1}{2}})] = -\frac{2}{\sqrt{\pi}} B(-\nu)\tilde{B}(\frac{1}{2})t^{\nu/2}J_\nu(2t^{\frac{1}{2}})$

56. $J_{-\nu}(t^{\frac{1}{2}})Y_{-\nu}(t^{\frac{1}{2}}) = -\frac{1}{\sqrt{\pi}} B(-\nu)\tilde{B}(\frac{1}{2})t^{\nu/2}J_{-\nu}(2t^{\frac{1}{2}})$

57. $[\cos(\nu\pi)J_{\nu-\mu}(t^{\frac{1}{2}})-\sin(\nu\pi)Y_{\nu-\mu}(t^{\frac{1}{2}})] = 2^{-\mu}B(\frac{\nu-\mu}{2})\tilde{B}(\frac{\nu+\mu}{2})t^{\mu/2}J_{-\nu}(t^{\frac{1}{2}})$

58. $[J_\nu(t^{\frac{1}{2}})J_{-\nu}(t^{\frac{1}{2}})-Y_{-\nu}(t^{\frac{1}{2}})Y_{-\nu}(t^{\frac{1}{2}})] = \frac{2}{\sqrt{\pi}} B(-\nu)\tilde{B}(\frac{1}{2})t^{\nu/2}J_\nu(2t^{\frac{1}{2}})$

59. $[\cos(\nu\pi)Y_{-\mu-\nu}(t^{\frac{1}{2}})-\sin(\pi\nu)J_{-\mu-\nu}(t^{\frac{1}{2}})] = 2^{-\mu}B(-\frac{\mu+\nu}{2})\tilde{B}(\frac{\mu-\nu}{2})t^{\mu/2}Y_\nu(t^{\frac{1}{2}})$

60. $H_{\nu}^{(j)}(t^{\frac{1}{2}}) = 2^{\mu}B(\frac{\nu}{2})\tilde{B}(\frac{\nu}{2} - \mu)t^{-\mu/2}H_{\nu-\mu}^{(j)}(t^{\frac{1}{2}})$ $\quad j=1,2$

61. $H_{-\nu}^{(j)}(t^{\frac{1}{2}}) = i\sqrt{\pi}\ 2^{\nu-1}B(\frac{1-\nu}{2})\tilde{B}(-\ \frac{3\nu}{2})t^{-\nu/2}[H_{-\nu}^{(j)}(\frac{t^{\frac{1}{2}}}{2})]^{2}$ $\quad j=1,2$

62. $H_{\nu}^{(j)}(t^{\frac{1}{2}}) = i\sqrt{\pi}\ 2^{\nu-1}B(\frac{1-\nu}{2})\tilde{B}(-\ \frac{3\nu}{2})t^{-\nu/2}[H_{\nu}^{(j)}(\frac{t^{\frac{1}{2}}}{2})H_{-\nu}^{(j)}(\frac{t^{\frac{1}{2}}}{2})]$ $\quad j=1,2$

63. $H_{\nu}^{(j)}(t^{-\frac{1}{2}}) = 2^{\mu}A(\frac{\nu}{2} - 1)\tilde{A}(\frac{\nu}{2} - \mu - 1)t^{\mu/2}H_{\nu-\mu}^{(j)}(t^{-\frac{1}{2}})$ $\quad j=1,2$

64. $H_{-\nu}^{(j)}(t^{-\frac{1}{2}}) = i\sqrt{\pi}\ 2^{\nu-1}A(-\ \frac{\nu+1}{2})\tilde{A}(-\ \frac{3\nu}{2} - 1)t^{\nu/2}[H_{-\nu}^{(j)}(\frac{1}{2t^{\frac{1}{2}}})]^{2}$ $\quad j=1,2$

65. $H_{\nu}^{(j)}(t^{-\frac{1}{2}}) = i\sqrt{\pi}\ 2^{\nu-1}A(-\ \frac{\nu+1}{2})\tilde{A}(-\ \frac{3\nu}{2} - 1)t^{\nu/2}[H_{\nu}^{(j)}(\frac{1}{2t^{\frac{1}{2}}})][H_{-\nu}^{(j)}(\frac{1}{2t^{\frac{1}{2}}})]$ $\quad j=1,2$

<u>Modified Bessel functions</u>

1. $I_\nu(t) = \frac{2^\nu}{\sqrt{\pi}} e^{-t} A(-\frac{1}{2})\tilde{A}(\nu) t^\nu e^{2t}$

2. $I_\nu(t^{-1}) = \frac{2^\nu}{\sqrt{\pi}} e^{-1/t} B(\frac{1}{2})\tilde{B}(\nu+1) t^{-\nu} e^{2/t}$

3. $I_\nu(t^{\frac{1}{2}}) = 2^{-\nu}\tilde{A}(\frac{\nu}{2})\tilde{M} t^{\nu/2} e^{t/4}$

4. $I_\nu(t^{-\frac{1}{2}}) = 2^{-\nu}\tilde{B}(\frac{\nu}{2} + 1)\tilde{N} t^{-\nu/2} e^{1/4t}$

5. $[I_\nu(t^{\frac{1}{2}})]^2 = \frac{1}{\sqrt{\pi}} A(-\frac{1}{2})\tilde{A}(\nu) t^{\nu/2} I_\nu(2t^{\frac{1}{2}})$

6. $[I_\nu(t^{\frac{1}{2}})]^2 = \frac{1}{\sqrt{\pi}} A(-\frac{1}{2})\tilde{A}(\nu)\tilde{M} t^\nu e^t$

7. $[I_\nu(t^{-\frac{1}{2}})]^2 = \frac{1}{\sqrt{\pi}} B(\frac{1}{2})\tilde{B}(\nu+1) t^{-\nu/2} I_\nu(2t^{-\frac{1}{2}})$

8. $[I_\nu(t^{-\frac{1}{2}})]^2 = \frac{1}{\sqrt{\pi}} B(\frac{1}{2})\tilde{B}(\nu+1)\tilde{B}(1)\tilde{N} t^{-\nu} e^{1/t}$

9. $I_{\nu/2}(t) = \frac{e^{-t}}{\sqrt{\pi}} A(-\frac{1}{2}) M I_\nu(2\sqrt{2}\, t^{\frac{1}{2}})$

10. $I_{\nu/2}(t^{-1}) = \frac{1}{\sqrt{\pi}} e^{-1/t} B(\frac{1}{2}) N I_\nu(2\sqrt{2}\, t^{-\frac{1}{2}})$

11. $I_{\mu+\nu}(t^{\frac{1}{2}}) = 2^{-\mu} A(\frac{\nu-\mu}{2})\tilde{A}(\frac{\mu+\nu}{2}) t^{\mu/2} I_\nu(t^{\frac{1}{2}})$

12. $I_{2\nu}(t^{\frac{1}{2}}) = 2^{-\nu}\widetilde{A}(\nu)t^{\nu/2}I_{\nu}(t^{\frac{1}{2}})$

13. $I_{\mu+\nu}(t^{-\frac{1}{2}}) = 2^{-\mu}B(\frac{\nu-\mu+2}{2})B(\frac{\mu+\nu+2}{2})t^{-\mu/2}I_{\nu}(t^{-\frac{1}{2}})$

14. $I_{2\nu}(t^{-\frac{1}{2}}) = 2^{-\nu}B(1)\widetilde{B}(\nu+1)t^{-\nu/2}I_{\nu}(t^{-\frac{1}{2}})$

15. $I_{\nu}(2\alpha\beta t) = e^{(\alpha^2+\beta^2)t}MJ_{\nu}(2\alpha t^{\frac{1}{2}})J_{\nu}(2\beta t^{\frac{1}{2}})$

16. $I_{\nu}(2\alpha\beta t^{-1}) = e^{(\alpha^2+\beta^2)/t}B(1)NJ_{\nu}(2\alpha t^{-\frac{1}{2}})J_{\nu}(2\beta t^{-\frac{1}{2}})$

17. $I_{\nu}(t) = \frac{e^{\mp t}2^{-\nu}}{\Gamma(\nu+1)\Gamma(\lambda+\nu)} A(\lambda-1)Mt^{\nu}{}_1F_2(\nu + \frac{1}{2},2\nu+1;\lambda+\nu;\pm 2t)$

18. $I_{\nu}(t^{-1}) = \frac{e^{\mp(1/t)}2^{-\nu}}{\Gamma(\nu+1)\Gamma(\lambda+\nu)} B(\lambda)Nt^{-\nu}{}_1F_2(\nu + \frac{1}{2},2\nu+1;\lambda+\nu;\pm 2t^{-1})$

19. $I_{\nu}(t) = \frac{2^{-\nu}}{\Gamma(\nu+1)\Gamma(\lambda+\nu)} A(\lambda-1)Mt^{\nu}{}_0F_3(\nu+1,\frac{\lambda+\nu}{2},\frac{\lambda+\nu+1}{2};(\frac{t}{4})^2)$

20. $I_{\nu}(t^{-1}) = \frac{2^{-\nu}}{\Gamma(\nu+1)\Gamma(\lambda+\nu)} B(\lambda)Nt^{-\nu}{}_0F_3(\nu+1,\frac{\lambda+\nu}{2},\frac{\lambda+\nu+1}{2};(\frac{1}{4t})^2)$

21. $I_{\nu}(t) = \frac{\Gamma(\nu+\frac{1}{2})e^{-t}}{\sqrt{\pi}} 2^{\nu}\widetilde{A}(\nu)\widetilde{M}t^{\nu}(1-2t)^{-\nu-\frac{1}{2}}$

22. $I_{\nu}(t^{-1}) = \frac{\Gamma(\nu+\frac{1}{2})e^{-1/t}}{\sqrt{\pi}} 2^{\nu}\widetilde{B}(\nu+1)\widetilde{N}t^{\frac{1}{2}}(t-2)^{-\nu-\frac{1}{2}}$

23. $I_{\nu}(t) = \frac{2^{\nu}\Gamma(\nu+\frac{1}{2})}{\sqrt{\pi}\ \Gamma(\mu+2\nu+1)} e^{\mp t}A(\mu+\nu)\widetilde{A}(\nu)t^{\nu}{}_1F_1(\nu+ \frac{1}{2};\mu+2\nu+1;\pm 2t)$

24. $I_\nu(t^{-1}) = \dfrac{2^\nu \Gamma(\nu+\frac{1}{2})}{\sqrt{\pi}\ \Gamma(\mu+2\nu+1)}\ e^{\mp 1/t} B(\mu+\nu+1)\tilde{B}(\nu+1)\, t^{-\nu}\ {}_1F_1(\nu+\frac{1}{2};\mu+2\nu+1;\pm\frac{2}{t})$

25. $I_\nu(t) = \dfrac{2^{-\nu}\Gamma(\lambda+\nu)}{\Gamma(\nu+1)\Gamma(\lambda+\mu+\nu)}\ e^{\mp t} A(\mu+\lambda-1)\tilde{A}(\lambda-1)\, t^{\nu}\ {}_2F_2(\nu+\frac{1}{2},\lambda+\nu;2\nu+1,\mu+\lambda+\nu,\pm 2t)$

26. $I_\nu(\frac{1}{t}) = \dfrac{2^{-\nu}\Gamma(\lambda+\nu)}{\Gamma(\nu+1)\Gamma(\lambda+\mu+\nu)}\ e^{\mp 1/t} B(\mu+\lambda)\tilde{B}(\lambda)\, t^{-\nu}\ {}_2F_2(\nu+\frac{1}{2},\lambda+\nu;2\nu+1,\mu+\lambda+\nu;\pm\frac{2}{t})$

27. $I_\nu(t^{\frac{1}{2}})I_{\nu-1}(t^{\frac{1}{2}}) = \dfrac{1}{\sqrt{\pi}}\tilde{A}(\nu-\frac{1}{2})\, t^{(\nu-1)/2} I_\nu(2t^{\frac{1}{2}})$

28. $I_\nu(t^{\frac{1}{2}})I_{-\nu}(t^{\frac{1}{2}}) = \dfrac{1}{\sqrt{\pi}}\tilde{A}(-\nu)A(-\frac{1}{2})\, t^{-\nu/2} I_\nu(t^{\frac{1}{2}})$

29. $I_\nu(t) = \dfrac{2^{\nu-\mu}\Gamma(\frac{1}{2}-\mu+\nu)}{\sqrt{\pi}\ \Gamma(1-\mu+2\nu)}\ e^{t} B(\nu)\tilde{B}(\nu-\mu)\, t^{\nu-\mu}\ {}_1F_1(\frac{1}{2}-\mu+\nu,1-\mu+2\nu;-2t)$

30. $I_\nu(t^{-1}) = \dfrac{2^{\nu-\mu}\Gamma(\frac{1}{2}-\mu+\nu)}{\sqrt{\pi}\ \Gamma(1-\mu+2\nu)}\ e^{1/t} A(\nu-1)\tilde{A}(\nu-\mu-1)\, t^{\mu-\nu}\ {}_1F_1(\frac{1}{2}-\mu+\nu,1-\mu+2\nu;-\frac{2}{t})$

31. $I_\nu(t^{\frac{1}{2}})K_\nu(t^{\frac{1}{2}}) = \dfrac{1}{\sqrt{\pi}}A(-\frac{1}{2})\tilde{A}(\nu)\, t^{\nu/2} K_\nu(2t^{\frac{1}{2}})$

32. $I_\nu(t^{-\frac{1}{2}})K_\nu(t^{-\frac{1}{2}}) = \dfrac{1}{\sqrt{\pi}}B(\frac{1}{2})\tilde{B}(\nu+1)\, t^{-\nu/2} K_\nu(2t^{-\frac{1}{2}})$

33. $K_\nu(t) = \sqrt{\pi}\ 2^\nu e^t B(-\nu)\tilde{B}(\frac{1}{2})\, t^\nu e^{-2t}$

34. $K_\nu(t^{-1}) = \sqrt{\pi}\ 2^\nu e^{1/t} A(-\nu-1)\tilde{A}(-\frac{1}{2})\, t^{-\nu} e^{-2/t}$

35. $K_\nu(t^{\frac{1}{2}}) = \sqrt{\pi}\ 2^{-\nu-1} B(-\frac{\nu}{2})\tilde{B}(\frac{\nu+1}{2})\, t^{\nu/2} e^{-t^{\frac{1}{2}}}$

36. $K_\nu(t^{\frac{1}{2}}) = \sqrt{\pi}\; 2^{-\nu} B(-\frac{\nu}{2})\tilde{B}(\frac{\nu-1}{2}) t^{(\nu-1)/2} e^{-t^{\frac{1}{2}}}$

37. $K_\nu(t^{-\frac{1}{2}}) = \sqrt{\pi}\; 2^{-\nu-1} A(-\frac{\nu}{2} - 1)\tilde{A}(\frac{\nu-1}{2}) t^{-\nu/2} e^{-t^{-\frac{1}{2}}}$

38. $K_\nu(t^{-\frac{1}{2}}) = \sqrt{\pi}\; 2^{-\nu} A(-\frac{\nu}{2} - 1)\tilde{A}(\frac{\nu-3}{2}) t^{(1-\nu)/2} e^{-t^{-\frac{1}{2}}}$

39. $K_\nu(t) = \dfrac{\sqrt{\pi}\; e^t}{\sqrt{2}} B(\mu + \frac{1}{2})\tilde{B}(\frac{1}{2}) t^{-\frac{1}{2}} e^{-t} W_{-\mu,\nu}(2t)$

40. $K_\nu(t^{-1}) = \dfrac{\sqrt{\pi}\; e^{1/t}}{\sqrt{2}} A(\mu - \frac{1}{2})\tilde{A}(-\frac{1}{2}) t^{\frac{1}{2}} e^{-1/t} W_{-\mu,\nu}(\frac{2}{t})$

41. $K_\nu(t) = \dfrac{\sqrt{\pi}\; 2^{-(\mu+1)/2}\Gamma(\frac{1}{2}-\mu+\nu)}{\Gamma(\frac{1}{2}+\nu)}\; e^{-t} B(\nu)\tilde{B}(\nu-\mu) e^t t^{-(\mu+1)/2} W_{\frac{\mu}{2},\nu-\frac{\mu}{2}}(2t)$

42. $K_\nu(t^{-1}) = \dfrac{\sqrt{\pi}\; 2^{-(\mu+1)/2}\Gamma(\frac{1}{2}-\mu+\nu)}{\Gamma(\nu+\frac{1}{2})}\; e^{-1/t} A(\nu-1)\tilde{A}(\nu-\mu-1) e^{1/t} t^{(\mu+1)/2} W_{\frac{\mu}{2},\nu-\frac{\mu}{2}}(\frac{2}{t})$

43. $K_{2\nu}(t^{\frac{1}{2}}) = \dfrac{1}{\sqrt{\pi}} B(\frac{1}{2})[K_\nu(\frac{1}{2}\, t^{\frac{1}{2}})]^2$

44. $K_{2\nu}(t^{-\frac{1}{2}}) = \dfrac{1}{\sqrt{\pi}} A(-\frac{1}{2})\tilde{A}(-\frac{1}{2})[K_\nu(\frac{1}{2}\, t^{\frac{1}{2}})]^2$

45. $K_\nu(2t^{\frac{1}{2}}) = \dfrac{1}{\sqrt{\pi}} B(\frac{1-\nu}{2})\tilde{B}(-\frac{3\nu}{2}) t^{-\nu/2}[K_\nu(t^{\frac{1}{2}})]^2$

46. $K_\nu(2t^{-\frac{1}{2}}) = \dfrac{1}{\sqrt{\pi}} A(-\frac{\nu+1}{2})\tilde{A}(-\frac{3\nu}{2} - 1) t^{\nu/2}[K_\nu(t^{-\frac{1}{2}})]^2$

47. $K_\nu(2t^{\frac{1}{2}}) = \dfrac{1}{\sqrt{\pi}} B(\frac{1-\nu}{2})\tilde{B}(1 - \frac{3\nu}{2}) t^{(1-\nu)/2} K_\nu(t^{\frac{1}{2}}) K_{\nu-1}(t^{\frac{1}{2}})$

48. $K_\nu(2t^{-\frac{1}{2}}) = \frac{1}{\sqrt{\pi}} A(-\frac{\nu+1}{2})\tilde{A}(-\frac{3\nu}{2})t^{(\nu-1)/2}K_\nu(t^{-\frac{1}{2}})K_{\nu-1}(t^{-\frac{1}{2}})$

49. $K_\nu(t^{\frac{1}{2}}) = 2^\mu B(\frac{\nu}{2})\tilde{B}(\frac{\nu}{2} - \mu)t^{-\frac{1}{2}} K_{\nu-\mu}(t^{\frac{1}{2}})$

50. $K_\nu(t^{-\frac{1}{2}}) = 2^\mu A(\frac{\nu}{2} - 1)\tilde{A}(\frac{\nu}{2} - \mu - 1)t^{\mu/2}K_{\nu-\mu}(t^{-\frac{1}{2}})$

51. $K_\nu(2t^{\frac{1}{2}}) = \frac{1}{\sqrt{\pi}} B(\frac{\nu+1}{2})\tilde{B}(\frac{3\nu}{2})t^{\nu/2}[K_\nu(t^{\frac{1}{2}})]^2$

52. $K_\nu(2t^{-\frac{1}{2}}) = \frac{1}{\sqrt{\pi}} A(\frac{\nu-1}{2})\tilde{A}(\frac{3\nu}{2} - 1)t^{-\nu/2}[K_\nu(t^{-\frac{1}{2}})]^2$

53. $K_\nu(t^{\frac{1}{2}}) = 2^{-\nu-1}B(-\frac{\nu}{2})Nt^{\nu/2}e^{-t/4}$

54. $K_\nu(t^{-\frac{1}{2}}) = 2^{-\nu-1}A(-\frac{\nu}{2} - 1)Mt^{-\nu/2}e^{-1/4t}$

Struve's functions

1. $\mathbf{H}_\nu(t^{\frac{1}{2}}) = \frac{2^{-\nu}}{\sqrt{\pi}} A(-\frac{\nu+1}{2})\tilde{A}(\frac{\nu}{2})t^{\nu/2}\sin(t^{\frac{1}{2}})$

2. $\mathbf{H}_\nu(t^{\frac{1}{2}}) = 2^{-\nu-1}A(-\frac{\nu+1}{2})\tilde{A}(\frac{\nu}{2})\tilde{A}(-\frac{\nu}{2})\tilde{M}t^{(\nu+1)/2}e^{-t/4}$

3. $\mathbf{H}_\nu(t^{\frac{1}{2}}) = \frac{2^{1-\nu}}{\sqrt{\pi}} A(\frac{1-\nu}{2})\tilde{A}(\frac{\nu}{2})t^{(\nu-1)/2}(1-\cos t^{\frac{1}{2}})$

4. $\mathbf{H}_{\mu+\nu}(t^{\frac{1}{2}}) = 2^{-\mu}A(\frac{\nu-\mu}{2})\tilde{A}(\frac{\mu+\nu}{2})t^{\mu/2}\mathbf{H}_\nu(t^{\frac{1}{2}})$

5. $\mathbf{H}_{2\nu}(t^{\frac{1}{2}}) = 2^{-\nu}\tilde{A}(\nu)t^{\nu/2}\mathbf{H}_\nu(t^{\frac{1}{2}})$

6. $\mathbf{H}_{2\nu}(t^{\frac{1}{2}}) = \frac{2^{1-2\nu}}{\sqrt{\pi}} A(\frac{1}{2} - \nu)\tilde{A}(\nu)t^{\nu-\frac{1}{2}}(1-\cos t^{\frac{1}{2}})$

7. $\mathbf{H}_\nu(t^{\frac{1}{2}}) = \frac{2^{-\nu}\Gamma(\nu)}{\sqrt{\pi}\,\Gamma(\nu + \frac{3}{2})\Gamma(\lambda+\mu)} A(\lambda+\mu - \frac{\nu+3}{2})\tilde{A}(\lambda - \frac{\nu+3}{2}) \cdot$

$$\cdot\, t^{(\nu+1)/2}{}_2F_3(1,\lambda;\tfrac{3}{2},\nu + \tfrac{3}{2},\lambda+\mu;- \tfrac{t}{4})$$

8. $\mathbf{H}(2t^{\frac{1}{2}}) = \sqrt{\pi}\, B(\frac{\nu+1}{2})\tilde{B}(\frac{3\nu}{2})t^{\nu/2}[J_\nu(t^{\frac{1}{2}})]^2$

9. $\mathbf{H}(t^{-\frac{1}{2}}) = \frac{2^{-\nu}}{\sqrt{\pi}} B(\frac{1-\nu}{2})\tilde{B}(\frac{\nu}{2} + 1)t^{-\nu/2}\sin(t^{-\frac{1}{2}})$

10. $\mathbf{H}(t^{-\frac{1}{2}}) = \frac{2^{1-\nu}}{\sqrt{\pi}} B(\frac{3-\nu}{2})\tilde{B}(\frac{\nu}{2} + 1)t^{(1-\nu)/2}(1-\cos t^{-\frac{1}{2}})$

11. $\mathbf{H}_{\mu+\nu}(t^{-\frac{1}{2}}) = 2^{-\mu}B(\frac{\nu-\mu}{2} + 1)\widetilde{B}(\frac{\mu+\nu}{2} + 1)t^{-\mu/2}\mathbf{H}_{\nu}(t^{-\frac{1}{2}})$

12. $\mathbf{H}_{2\nu}(t^{-\frac{1}{2}}) = 2^{-\nu}B(1)\widetilde{B}(\nu+1)t^{-\nu/2}\mathbf{H}_{\nu}(t^{-\frac{1}{2}})$

13. $\mathbf{H}_{2\nu}(t^{-\frac{1}{2}}) = \dfrac{2^{1-2\nu}}{\sqrt{\pi}} B(\frac{3}{2} - \nu)\widetilde{B}(\nu+1)t^{\frac{1}{2}-\nu}(1-\cos t^{-\frac{1}{2}})$

14. $\mathbf{H}_{\nu}(t^{-\frac{1}{2}}) = 2^{-\nu-1}B(\frac{1-\nu}{2})\widetilde{B}(\frac{\nu+1}{2})\widetilde{B}(1 - \frac{\nu}{2})\widetilde{N}t^{-(\nu+1)/2}e^{-1/4t}$

15. $\mathbf{L}_{\nu}(t^{\frac{1}{2}}) = \dfrac{\Gamma(\lambda)2^{-\nu}}{\sqrt{\pi}\ \Gamma(\nu + \frac{3}{2})\Gamma(\lambda+\mu)} A(\mu+\lambda- \frac{\nu+3}{2})\widetilde{A}(\lambda - \frac{\nu+3}{2}) \cdot$

$\cdot\ t^{(\nu+1)/2}{}_2F_3(1,\lambda;\frac{3}{2},\nu + \frac{3}{2},\lambda+\mu;\frac{t}{4})$

16. $\mathbf{L}_{\mu+\nu}(t^{\frac{1}{2}}) = 2^{-\mu}A(\frac{\nu-\mu}{2})\widetilde{A}(\frac{\nu+\mu}{2})t^{\mu/2}\mathbf{L}_{\nu}(t^{\frac{1}{2}})$

17. $\mathbf{L}_{2\nu}(t^{\frac{1}{2}}) = 2^{-\nu}\widetilde{A}(\nu)t^{\nu/2}\mathbf{L}_{\nu}(t^{\frac{1}{2}})$

18. $\mathbf{L}_{\nu}(t^{-\frac{1}{2}}) = \dfrac{\Gamma(\lambda)2^{-\nu}}{\sqrt{\pi}\ \Gamma(\nu + \frac{3}{2})\Gamma(\lambda+\mu)} B(\mu+\lambda- \frac{\nu+1}{2})\widetilde{B}(\lambda - \frac{\nu+1}{2}) \cdot$

$\cdot\ t^{-(\nu+1)/2}{}_2F_3(1,\lambda;\frac{3}{2},\nu + \frac{3}{2},\lambda+\mu;\frac{1}{4t})$

19. $\mathbf{L}_{\mu+\nu}(t^{-\frac{1}{2}}) = 2^{-\mu}B(\frac{\nu-\mu+2}{2})\widetilde{B}(\frac{\nu+\mu+2}{2})t^{-\mu/2}\mathbf{L}_{\nu}(t^{-\frac{1}{2}})$

20. $\mathbf{L}_{2\nu}(t^{-1/2}) = 2^{-\nu}B(1)\widetilde{B}(\nu+1)t^{-\nu/2}\mathbf{L}_{\nu}(t^{-1/2})$

21. $[\mathbf{H}_{\nu}(2t^{1/2}) - 2Y_{\nu}(2t^{1/2})] = \sqrt{\pi}\, B(\frac{\nu+1}{2})\widetilde{B}(\frac{3\nu}{2})t^{\nu/2}[Y_{\nu}(t^{1/2})]^2$

22. $[\cos(\nu\pi)\mathbf{H}_{\mu+\nu}(t^{1/2}) + \sin(\mu\pi)J_{-\mu-\nu}(t^{1/2})] = 2^{-\mu}\cos[(\mu+\nu)\pi]B(-\frac{\mu+\nu}{2})\widetilde{B}(\frac{\mu-\nu}{2}) \cdot$

$\cdot\, t^{\mu/2}\mathbf{H}_{\nu}(t^{1/2})$

23. $[\mathbf{H}_{-\mu}(t^{1/2}) - Y_{-\mu}(t^{1/2})] = \frac{2^{\mu+1}}{\pi} B(-\frac{\mu}{2})\widetilde{B}(-\frac{3\mu}{2})t^{-\mu/2}S_{0,2\mu}(t^{1/2})$

24. $[\mathbf{H}_{\mu+\nu}(t^{1/2}) - Y_{\mu+\nu}(t^{1/2})] = \frac{2^{-\mu}\cos[(\mu+\nu)\pi]}{\cos(\nu\pi)} B(-\frac{\mu+\nu}{2})\widetilde{B}(\frac{\mu-\nu}{2})t^{\mu/2} \cdot$

$\cdot\, [\mathbf{H}_{\nu}(t^{1/2}) - Y_{\nu}(t^{1/2})]$

25. $[I_{\nu}(t^{1/2}) + \mathbf{L}_{\nu}(t^{1/2})] = \frac{2^{-\nu}}{\sqrt{\pi}} A(-\frac{\nu+1}{2})\widetilde{A}(\frac{\nu}{2})t^{\nu/2}e^{t^{1/2}}$

26. $[I_{\nu}(t^{-1/2}) + \mathbf{L}_{\nu}(t^{-1/2})] = \frac{2^{-\nu}}{\sqrt{\pi}} B(\frac{1-\nu}{2})\widetilde{B}(\frac{\nu}{2} + 1)t^{-\nu/2}e^{t^{-1/2}}$

27. $[I_{\nu}(t^{1/2}) + \mathbf{L}_{\nu}(t^{1/2})] = \frac{2^{1-\nu}}{\sqrt{\pi}} A(\frac{1-\nu}{2})\widetilde{A}(\frac{\nu}{2})t^{(\nu-1)/2}\{e^{t^{1/2}} - 1\}$

28. $[I_{\nu}(t^{-1/2}) + \mathbf{L}_{\nu}(t^{-1/2})] = \frac{2^{1-\nu}}{\sqrt{\pi}} B(\frac{3-\nu}{2})\widetilde{B}(\frac{\nu}{2} + 1)t^{(1-\nu)/2}\{e^{t^{-1/2}} - 1\}$

29. $[I_{\nu}(2t^{1/2}) - \mathbf{L}_{\nu}(2t^{1/2})] = \frac{2}{\sqrt{\pi}} B(\frac{\nu+1}{2})\widetilde{B}(\frac{3\nu}{2})t^{\nu/2}I_{\nu}(t^{1/2})K_{\nu}(t^{1/2})$

30. $$[I_{-\mu-\nu}(2t^{\frac{1}{2}}) - \mathbf{L}_{\mu+\nu}(t^{\frac{1}{2}})] = \frac{2^{-\mu}\cos[(\mu+\nu)\pi]}{\cos(\nu\pi)} B(-\frac{\mu+\nu}{2})\tilde{B}(\frac{\mu-\nu}{2})t^{\mu/2} \cdot$$

$$\cdot\ [I_{-\nu}(t^{\frac{1}{2}}) - \mathbf{L}_{\nu}(t^{\frac{1}{2}})]$$

Lommel functions

1. $s_{\alpha,\beta}(t^{\frac{1}{2}}) = \frac{1}{4}\,\Gamma(\frac{\alpha+\beta+1}{2})\Gamma(\frac{\alpha-\beta+1}{2})A(-\,\frac{\alpha+1}{2})\tilde{A}(\frac{\beta}{2})\tilde{A}(-\,\frac{\beta}{2})\tilde{M}t^{(\alpha+1)/2}e^{-t/4}$

2. $s_{\mu+\nu-1,\mu-\nu}(t^{\frac{1}{2}}) = \Gamma(\mu)\Gamma(\nu)2^{\nu-2}A(-\,\frac{\mu+\nu}{2})\tilde{A}(\frac{\mu-\nu}{2})t^{\mu/2}J_{\nu}(t^{\frac{1}{2}})$

3. $s_{\alpha,\beta}(t^{\frac{1}{2}}) = \dfrac{\Gamma(\frac{\alpha+\beta+1}{2})\Gamma(\frac{\alpha-\beta+1}{2})}{4\sqrt{\pi}}\,A(-\,\frac{\alpha+1}{2})\tilde{A}(\frac{\beta}{2})A(-\,\frac{\alpha+2}{2})\tilde{A}(-\,\frac{\beta}{2})t^{(\alpha+1)/2}\cos(t^{\frac{1}{2}})$

4. $s_{\alpha,\beta}(t^{-\frac{1}{2}}) = \frac{1}{4}\,\Gamma(\frac{\alpha+\beta+1}{2})\Gamma(\frac{\alpha-\beta+1}{2})B(\frac{1-\alpha}{2})\tilde{B}(\frac{\beta}{2}+1)\tilde{B}(1-\frac{\beta}{2})\tilde{N}e^{-1/4t}$

5. $s_{\mu+\nu-1,\mu-\nu}(t^{-\frac{1}{2}}) = \Gamma(\mu)\Gamma(\nu)2^{\nu-2}B(1-\frac{\mu+\nu}{2})\tilde{B}(\frac{\mu-\nu}{2}+1)t^{-\mu/2}J_{\nu}(t^{-\frac{1}{2}})$

6. $s_{\alpha,\beta}(t^{-\frac{1}{2}}) = \dfrac{\Gamma(\frac{\alpha+\beta+1}{2})\cdot\Gamma(\frac{\alpha-\beta+1}{2})}{4\sqrt{\pi}}\,B(\frac{1-\alpha}{2})\tilde{B}(\frac{\beta}{2}+1)B(-\,\frac{\alpha}{2})\tilde{B}(1-\frac{\beta}{2})t^{-(\alpha+1)/2}\cos(t^{-\frac{1}{2}})$

7. $[2^{\mu}\mathrm{ctn}(\nu\pi)J_{\mu+\nu}(t^{\frac{1}{2}}) + \dfrac{2^{\nu+2}\Gamma(\nu+1)}{\pi\Gamma(\mu)}\,s_{\mu-\nu-1,\mu+\nu}(t^{\frac{1}{2}})] = A(\frac{\nu-\mu}{2})\tilde{A}(\frac{\mu+\nu}{2})t^{\mu/2}Y_{\nu}(t^{\frac{1}{2}})$

8. $S_{\lambda+\mu,\nu+\mu}(t^{\frac{1}{2}}) = \dfrac{\Gamma(\frac{1-\lambda-\nu}{2})}{\Gamma(\frac{1-\lambda-\nu}{2}-\mu)}\,B(-\,\frac{\mu+\nu}{2})\tilde{B}(\frac{\mu-\nu}{2})t^{\mu/2}S_{\lambda,\nu}(t^{\frac{1}{2}})$

9. $[2^{\mu}Y_{\mu+\nu}(t^{\frac{1}{2}}) + 2^{\nu+2}\,\dfrac{\Gamma(\nu+1)}{\pi\Gamma(\mu)}\,S_{\mu-\nu-1,\mu+\nu}(t^{\frac{1}{2}})] = A(\frac{\nu-\mu}{2})\tilde{A}(\frac{\nu-\mu}{2})t^{\mu/2}Y_{\nu}(t^{\frac{1}{2}})]$

10. $[Y_{-2\mu}(t^{\frac{1}{2}}) + \frac{2}{\pi}\,S_{0,2\mu}(t^{\frac{1}{2}})] = 2^{-\mu}B(-\mu)t^{\mu/2}\mathbf{H}_{-\mu}(t^{\frac{1}{2}})$

Gauss hypergeometric function

1. $F(a,b;c;-t) = \frac{\Gamma(c)}{\Gamma(b)} A(b-1)\widetilde{A}(c-1)(1+t)^{-a}$

2. $F(a,b;c;-t) = \frac{\Gamma(c)}{\Gamma(a)} A(a-1)\widetilde{A}(c-1)(1+t)^{-b}$

3. $F(a,b;c;-t) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} A(a-1)\widetilde{A}(c-1)A(b-1)Me^{-t}$

4. $F(a,b;c;-t) = \frac{\Gamma(c)\Gamma(c)}{\Gamma(a)\Gamma(b)} A(a-1)\widetilde{A}(c-1)A(b-1)\widetilde{A}(c-1)(1+t)^{-c}$

5. $F(a,b;c;-t^{-1}) = \frac{\Gamma(c)}{\Gamma(b)} B(b)\widetilde{B}(c)t^{a}(t+1)^{-a}$

6. $F(a,b;c;-t^{-1}) = \frac{\Gamma(c)}{\Gamma(a)} B(a)\widetilde{B}(c)t^{b}(t+1)^{-b}$

7. $F(a,b;c;-t^{-1}) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} B(a)\widetilde{B}(c)B(b)Ne^{-1/t}$

8. $F(a,b;c;-t^{-1}) = \frac{\Gamma(c)\Gamma(c)}{\Gamma(a)\Gamma(a)} B(a)\widetilde{B}(c)B(b)\widetilde{B}(c)t^{c}(t+1)^{-c}$

9. $.(1-t)^{c-1}F(a,b;c;1-t) = \frac{\Gamma(c)}{\Gamma(a)} \widetilde{B}(c-a)t^{c-a-b}.(1-t)^{a-1}$

10. $.(1-t)^{c-1}F(a,b;c;1-t) = \frac{\Gamma(c)}{\Gamma(b)} \widetilde{B}(c-b)t^{c-a-b}.(1-t)^{b-1}$

11. $.(1-t)^{c-1}F(a,b;c;1-t) = \Gamma(c)\widetilde{B}(c-a)\widetilde{B}(c-b)\widetilde{N}t^{c-a-b}e^{-t}$

12. $.(1-t)^{c-1}F(a,b;c;1-t) = \Gamma(c)B(c-a-b)\tilde{B}(c-a)\tilde{B}(c-b)\tilde{N}e^{-t}$

13. $.(t-1)^{c-1}F(a,b;c;1-t^{-1}) = \frac{\Gamma(c)}{\Gamma(c-a)}\, A(-a-b)\tilde{A}(-b)t^{a}.(t-1)^{c-a-1}$

14. $.(t-1)^{c-1}F(a,b;c;1-t^{-1}) = \frac{\Gamma(c)}{\Gamma(c-b)}\, A(-a-b)\tilde{A}(-a)t^{b}.(t-1)^{c-b-1}$

15. $.(t-1)^{c-1}F(a,b;c;1-t^{-1}) = \Gamma(c)A(-a-b)\tilde{A}(-a)\tilde{A}(-b)\tilde{M}t^{c-1}e^{-1/t}$

16. $.(t-1)^{c-1}F(a,b;c;1-t^{-1}) = \Gamma(c)A(c)\tilde{A}(-a)\tilde{A}(-b)\tilde{M}t^{a+b-1}e^{-1/t}$

17. $F(a,b;c;t^{-1}) = \frac{1}{\Gamma(d)}\, B(d)N\,{}_2F_2(a,b;c,d;t^{-1})$

18. $F(a,b;c;-t^{-1}) = \frac{1}{\Gamma(b)}\, B(b)N\,{}_1F_1(a;c;-t^{-1})$

19. $F(a,b;c;t) = \frac{1}{\Gamma(d)}\, A(d-1)M\,{}_2F_2(a,b;c,d;t)$

20. $F(a,b;c;-t) = \frac{1}{\Gamma(b)}\, A(b-1)M\,{}_1F_1(a;c;-t)$

<u>Generalized hypergeometric series</u>

1. $${}_0F_1(b;-t) = \Gamma(b)\tilde{A}(b-1)\tilde{M}e^{-t}$$

2. $${}_0F_1(b;-t) = \frac{\Gamma(b)}{\sqrt{\pi}} A(-\tfrac{1}{2})\tilde{A}(b-1)\cos(2t^{\frac{1}{2}})$$

3. $${}_0F_1(b;-t^{-1}) = \Gamma(b)\tilde{B}(b)\tilde{N}e^{-1/t}$$

4. $${}_0F_1(b;-t^{-1}) = \frac{\Gamma(b)}{\sqrt{\pi}} B(\tfrac{1}{2})\tilde{B}(b)\cos(2t^{-\frac{1}{2}})$$

5. $${}_1F_2(a;b_1,b_2;-t) = \frac{\Gamma(b_1)\Gamma(b_2)}{\Gamma(a)} A(a-1)\tilde{A}(b_1-1)\tilde{A}(b_2-1)\tilde{M}e^{-t}$$

6. $${}_1F_2(a;b_1,b_2;-t) = \frac{\Gamma(b_1)\Gamma(b_2)}{\sqrt{\pi}\,\Gamma(a)} A(a-1)\tilde{A}(b_1-1)A(-\tfrac{1}{2})\tilde{A}(b_2-1)\cos(2t^{\frac{1}{2}})$$

7. $${}_1F_2(a;b,\tfrac{3}{2};-t) = \frac{\Gamma(b)}{2\Gamma(a)} A(a-1)\tilde{A}(b-1)t^{-\frac{1}{2}}\sin(2t^{\frac{1}{2}})$$

8. $${}_1F_2(a;b,\tfrac{1}{2};-t) = \frac{\Gamma(b)}{\Gamma(a)} A(a-1)\tilde{A}(b-1)\cos(2t^{\frac{1}{2}})$$

9. $${}_1F_2(a;b,\nu+1;-t) = \frac{\Gamma(b)\Gamma(\nu+1)}{\Gamma(a)} A(a-1)\tilde{A}(b-1)t^{-\nu/2}J_\nu(2t^{\frac{1}{2}})$$

10. $${}_1F_2(a;b,\nu+1;t) = \frac{\Gamma(b)\Gamma(\nu+1)}{\Gamma(a)} A(a-1)\tilde{A}(b-1)t^{-\nu/2}I_\nu(2t^{\frac{1}{2}})$$

11. $${}_1F_2(a;b_1,b_2;-t^{-1}) = \frac{\Gamma(b_1)\Gamma(b_2)}{\Gamma(a)} B(a)\tilde{B}(b_1)\tilde{B}(b_2)\tilde{N}e^{-1/t}$$

12. $${}_1F_2(a;b_1,b_2;-t^{-1}) = \frac{\Gamma(b_1)\Gamma(b_2)}{\sqrt{\pi}\,\Gamma(a)} B(a)\widetilde{B}(b_1)B(\tfrac{1}{2})\widetilde{B}(b_2)\cos(2t^{-\frac{1}{2}})$$

13. $${}_1F_2(a;b,\tfrac{3}{2};-t^{-1}) = \frac{\Gamma(b)}{2\Gamma(a)} B(a)\widetilde{B}(b)t^{\frac{1}{2}}\sin(2t^{-\frac{1}{2}})$$

14. $${}_1F_2(a;b,\tfrac{1}{2};-t^{-1}) = \frac{\Gamma(b)}{\Gamma(a)} B(a)\widetilde{B}(b)\cos(2t^{-\frac{1}{2}})$$

15. $${}_1F_2(a;b,\nu+1;-t^{-1}) = \frac{\Gamma(b)\Gamma(\nu+1)}{\Gamma(a)} B(a)\widetilde{B}(b)t^{\nu/2}J_\nu(2t^{-\frac{1}{2}})$$

16. $${}_1F_2(a;b,\nu+1;t^{-1}) = \frac{\Gamma(b)\Gamma(\nu+1)}{\Gamma(a)} B(a)\widetilde{B}(b)t^{\nu/2}I_\nu(2t^{-\frac{1}{2}})$$

17. $${}_2F_2(a_1,a_2;b_1,b_2;-t) = \frac{\Gamma(b_1)\Gamma(b_2)}{\Gamma(a_1)\Gamma(a_2)} A(a_1-1)\widetilde{A}(b_1-1)A(a_2-1)\widetilde{A}(b_2-1)e^{-t}$$

18. $${}_2F_2(a_1,a_2;b_1,b_2;t) = \frac{\Gamma(b_1)\Gamma(b_2)}{\Gamma(a_1)\Gamma(a_2)} A(a_1-1)\widetilde{A}(b_1-1)A(a_1+a_2-2)\widetilde{A}(b_1+a_2-2)t^{1-a_2}e^t$$

19. $${}_2F_2(a_1,a_2;2a_1,b;t) = \frac{\Gamma(a_1+\frac{1}{2})\Gamma(b)}{\sqrt{\pi}\,\Gamma(a_2)} A(a_2-1)\widetilde{A}(b-1)A(a_1-1)\widetilde{A}(2a_1-1)e^t$$

20. $${}_2F_2(a_1,a_2;b_1b_2;t) = \frac{\Gamma(b_2)}{\Gamma(a_2)} A(a_2-1)\widetilde{A}(b_2-1)\,{}_1F_1(a_1;b;t)$$

21. $${}_2F_2(a_1,a_2;b_1,b_2;t) = \Gamma(b_2)\widetilde{A}(b_2-1)\widetilde{M}\,{}_2F_1(a_1,a_2;b_1;t)$$

22. $${}_2F_2(\nu+\tfrac{1}{2},a;2\nu+1,b;t) = \frac{\Gamma(\nu+1)\Gamma(b)}{\Gamma(a)} A(a-1)\widetilde{A}(b-1)t^{-\nu}e^{t/2}I_\nu(\tfrac{t}{2})$$

23. $${}_2F_2(\nu+\tfrac{1}{2},a;2\nu+1,b;\pm it) = \frac{\Gamma(\nu+1)\Gamma(b)}{\Gamma(a)} A(a-1)\widetilde{A}(b-1)t^{-\nu}e^{\pm it/2}J_\nu(\tfrac{t}{2})$$

24. $${}_2F_2(\alpha+n+1,a;\alpha+1,b;-t) = \frac{n!\Gamma(\alpha+1)\Gamma(b)}{\Gamma(\alpha+n+1)\Gamma(a)} A(a-1)\tilde{A}(b-1)e^{-t}L_n^\alpha(t)$$

25. $${}_2F_2(-n,a;\alpha+1,b;t) = \frac{n!\Gamma(\alpha+1)\Gamma(b)}{\Gamma(\alpha+n+1)\Gamma(a)} A(a-1)\tilde{A}(b-1)L_n^\alpha(t)$$

26. $${}_2F_2(a_1,a_2;b_1,b_2;t) = \frac{1}{\Gamma(b_3)} A(b_3-1)M_2F_3(a_1,a_2;b_1,b_2,b_3;t)$$

27. $${}_2F_2(a_1,a_2;b_1,b_2;t) = \Gamma(a_3)\tilde{A}(a_3-1)\tilde{M}_3F_2(a_1,a_2,a_3;b_1,b_2;t)$$

28. $${}_2F_2(a_1,a_2;b_1,b_2;t) = \frac{1}{\Gamma(a_2)} A(a_2-1)M_1F_2(a_1;b_1,b_2;t)$$

29. $${}_2F_2(a_1,a_2;b,\nu+1;-t) = \frac{\Gamma(b)\Gamma(\nu+1)}{\Gamma(a_1)\Gamma(a_2)} A(a_1-1)\tilde{A}(b-1)A(a_2-1)Mt^{-\nu/2}J_\nu(2t^{\frac{1}{2}})$$

30. $${}_2F_2(a_1,a_2;b_1,b_2;-t^{-1}) = \frac{\Gamma(b_1)\Gamma(b_2)}{\Gamma(a_1)\Gamma(a_2)} B(a_1)\tilde{B}(b_1)B(a_2)\tilde{B}(b_2)e^{-1/t}$$

31. $${}_2F_2(a_1,a_2;b_1,b_2;t^{-1}) = \frac{\Gamma(b_1)\Gamma(b_2)}{\Gamma(a_1)\Gamma(a_2)} B(a_1)\tilde{B}(b_1)B(a_1+a_2-1)\tilde{B}(b_1+a_2-1)t^{1-a_2}e^{1/t}$$

32. $${}_2F_2(a_1,a_2;2a_1,b;t^{-1}) = \frac{\Gamma(a_1+\frac{1}{2})\Gamma(b)}{\sqrt{\pi}\ \Gamma(a_2)} B(a_2)\tilde{B}(b)B(a_1)\tilde{B}(2a_1)e^{1/t}$$

33. $${}_2F_2(a_1,a_2;b_1,b_2;t^{-1}) = \frac{\Gamma(b_2)}{\Gamma(b_1)} B(a_2)\tilde{B}(b_2)\,{}_1F_1(a_1;b;t^{-1})$$

34. $${}_2F_2(a_1,a_2;b_1,b_2;t^{-1}) = \Gamma(b_2)\tilde{B}(b_2)\tilde{N}_2F_1(a_1,a_2;b;t^{-1})$$

35. $${}_2F_2(\nu+\frac{1}{2},a;2\nu+1,b;t^{-1}) = \frac{\Gamma(\nu+1)\Gamma(b)}{\Gamma(a)} B(a)\tilde{B}(b)t^\nu e^{1/2t}I_\nu(\frac{1}{2t})$$

36. $${}_2F_2(\alpha+n+1,a;\alpha+1,b;-t^{-1}) = \frac{n!\Gamma(\alpha+1)\Gamma(b)}{\Gamma(\alpha+n+1)\Gamma(a)} B(a)\widetilde{B}(b)e^{-1/t}L_n^{\alpha}(t^{-1})$$

37. $${}_2F_2(-n,a;\alpha+1,b;t^{-1}) = \frac{n!\Gamma(\alpha+1)\Gamma(b)}{\Gamma(\alpha+n+1)\Gamma(a)} B(a)\widetilde{B}(b)L_n^{\alpha}(t^{-1})$$

38. $${}_2F_2(a_1,a_2;b_1,b_2;t^{-1}) = \frac{1}{\Gamma(b_3)} B(b_3)N_2F_3(a_1,a_2;b_1,b_2,b_3;t^{-1})$$

39. $${}_2F_2(a_1,a_2;b_1,b_2;t^{-1}) = \Gamma(a_3)\widetilde{B}(a_3)\widetilde{N}_3F_2(a_1,a_2,a_3;b_1,b_2;t^{-1})$$

40. $${}_2F_2(a_1,a_2;b_1,b_2;t^{-1}) = \frac{1}{\Gamma(a_2)} B(a_2)N_1F_2(a_1;b_1,b_2;t^{-1})$$

41. $${}_2F_2(a_1,a_2;b,\nu+1;-t^{-1}) = \frac{\Gamma(b)\Gamma(\nu+1)}{\Gamma(a_1)\Gamma(a_2)} B(a_1)\widetilde{B}(b)B(a_2)Nt^{\nu/2}J_\nu(2t^{-\frac{1}{2}})$$

42. $${}_2F_3(a_1,a_2;b_1,b_2,b_3;-t) = \frac{\Gamma(b_1)\Gamma(b_2)\Gamma(b_3)}{\Gamma(a_1)\Gamma(a_2)} \cdot$$

$$\cdot\ A(a_1-1)\widetilde{A}(b_1-1)A(a_2-1)\widetilde{A}(b_2-1)\widetilde{A}(b_3-1)\widetilde{M}e^{-t}$$

43. $${}_2F_3(a,a+\tfrac{1}{2};\nu+1,b,b+\tfrac{1}{2};-t^2) = \frac{\Gamma(\nu+1)\Gamma(2b)}{\Gamma(2a)} A(2a-1)\widetilde{A}(2b-1)t^{-\nu}J_\nu(2t)$$

44. $${}_2F_3(1,a;\tfrac{\kappa+3-\nu}{2},\tfrac{\kappa+3+\nu}{2},b;-t) = \frac{(\kappa+1-\nu)(\kappa+1+\nu)\Gamma(b)}{\Gamma(a)} 2^{-\kappa}A(a-1)\widetilde{A}(b-1)t^{-(\kappa+1)/2}s_{\kappa,\nu}(2t^{\frac{1}{2}})$$

45. $${}_2F_3(1,a;\tfrac{3}{2},\nu+\tfrac{3}{2},b;-t) = \frac{\sqrt{\pi}\,\Gamma(\nu+3/2)\Gamma(b)}{2\Gamma(a)} A(a-1)\widetilde{A}(b-1)t^{-(\nu+1)/2}\mathbf{H}_\nu(2t^{\frac{1}{2}})$$

46. $${}_2F_3(\tfrac{1}{2},a;1+\nu,1-\nu,b;-t) = \frac{\Gamma(b)\Gamma(1+\nu)\Gamma(1-\nu)}{\Gamma(a)} A(a-1)\tilde{A}(b-1)J_\nu(t^{1/2})J_{-\nu}(t^{1/2})$$

47. $${}_2F_3(1,a;\tfrac{3}{2},\nu+\tfrac{3}{2},b;-t) = \frac{\Gamma(\nu+3/2)\Gamma(b)}{2\Gamma(a)} A(a-1)\tilde{A}(b-1)\tilde{A}(\nu+\tfrac{1}{2})t^{-1/2}\sin(2t^{1/2})$$

48. $${}_2F_3(1,a;\tfrac{3}{2},\nu+\tfrac{3}{2},b;t) = \frac{\sqrt{\pi}\,\Gamma(\nu+3/2)\Gamma(b)}{2\Gamma(a)} A(a-1)\tilde{A}(b-1)t^{-(\nu+1)/2}\mathbf{L}_\nu(2t^{1/2})$$

49. $${}_2F_3(a_1,a_2;b_1,b_2,b_3;t) = \Gamma(b_3)\tilde{A}(b_3-1)\tilde{M}\,{}_2F_2(a_1,a_2;b_1,b_2;t)$$

50. $${}_2F_3(a_1,a_2;b_1,b_2,b_3;t) = \frac{1}{\Gamma(b_4)} A(b_4-1)M\,{}_2F_4(a_1,a_2;b_1,b_2,b_3,b_4;t)$$

51. $${}_2F_3(a_1,a_2;b_1,b_2,b_3;t) = \frac{1}{\Gamma(a_2)} A(a_2-1)M\,{}_1F_3(a_1;b_1,b_2,b_3;t)$$

52. $${}_2F_3(a_1,a_2;b_1,b_2,b_3;t) = \Gamma(a_3)\tilde{A}(a_3-1)\tilde{M}\,{}_3F_3(a_1,a_2,a_3;b_1,b_2,b_3;t)$$

53. $${}_2F_3(a_1,a_2;b_1,b_2,b_3;-t^{-1}) = \frac{\Gamma(b_1)\Gamma(b_2)\Gamma(b_3)}{\Gamma(a_1)\Gamma(a_2)} B(a_1)\tilde{B}(b_1)B(a_2)\tilde{B}(b_2)\tilde{B}(b_3)\tilde{M}e^{-1/t}$$

54. $${}_2F_3(a,a+\tfrac{1}{2};\nu+1,b,b+\tfrac{1}{2};-t^{-2}) = \frac{\Gamma(\nu+1)\Gamma(2b)}{\Gamma(2a)} B(2a)\tilde{B}(2b)t^\nu J_\nu(\tfrac{2}{t})$$

55. $${}_2F_3(1,a;\tfrac{\kappa+3-\nu}{2},\tfrac{\kappa+3+\nu}{2},b;-t^{-1}) = \frac{(\kappa+1-\nu)(\kappa+1+\nu)\Gamma(b)}{\Gamma(a)} 2^{-\kappa}B(a)\tilde{B}(b)t^{(\kappa+1)/2}s_{\kappa,\nu}(2t^{-1/2})$$

56. $${}_2F_3(1,a;\tfrac{3}{2},\nu+\tfrac{3}{2},b;-t^{-1}) = \frac{\sqrt{\pi}\,\Gamma(\nu+3/2)\Gamma(b)}{2\Gamma(a)} B(a)\tilde{B}(b)t^{(\nu+1)/2}\mathbf{H}_\nu(2t^{-1/2})$$

57. $${}_2F_3(\tfrac{1}{2},a;1+\nu,1-\nu,b;-t^{-1}) = \frac{\Gamma(b)\Gamma(1+\nu)\Gamma(1-\nu)}{\Gamma(a)} B(a)\tilde{B}(b)J_\nu(t^{-1/2})J_{-\nu}(t^{-1/2})$$

58. $_2F_3(1,a;\frac{3}{2},\nu+\frac{3}{2},b;-t^{-1}) = \frac{\Gamma(\nu+3/2)\Gamma(b)}{2\Gamma(a)} B(a)\tilde{B}(b)B(1)\tilde{B}(\nu+\frac{3}{2})t^{\frac{1}{2}}\sin(2t^{-\frac{1}{2}})$

59. $_2F_3(1,a;\frac{3}{2},\nu+\frac{3}{2},b;t^{-1}) = \frac{\sqrt{\pi}\,\Gamma(\nu+3/2)\Gamma(b)}{2\Gamma(a)} B(a)\tilde{B}(b)t^{(\nu+1)/2}L_\nu(2t^{-\frac{1}{2}})$

60. $_2F_3(a_1,a_2;b_1,b_2,b_3;t^{-1}) = \Gamma(b_3)\tilde{B}(b_3)\tilde{N}_2F_2(a_1,a_2;b_1,b_2;t^{-1})$

61. $_2F_3(a_1,a_2;b_1,b_2,b_3;t^{-1}) = \frac{1}{\Gamma(b_4)} B(b_4)N_2F_4(a_1,a_2;b_1,b_2,b_3,b_4;t^{-1})$

62. $_2F_3(a_1,a_2;b_1,b_2,b_3;t^{-1}) = \frac{1}{\Gamma(a_2)} B(a_2)N_1F_3(a_1;b_1,b_2,b_3;t^{-1})$

63. $_2F_3(a_1,a_2;b_1,b_2,b_3;t^{-1}) = \Gamma(a_3)\tilde{B}(a_3)\tilde{N}_3F_3(a_1,a_2,a_3;b_1,b_2,b_3;t^{-1})$

64. $_3F_2(a_1,a_2,a_3;b_1,b_2;-t) = \frac{\Gamma(b_1)\Gamma(b_2)}{\Gamma(a_1)\Gamma(a_2)\Gamma(a_3)} A(a_1-1)\tilde{A}(b_1-1)A(a_2-1)\tilde{A}(b_2-1)A(a_3-1)Me^{-t}$

65. $_3F_2(a_1,a_2,a_2+\frac{1}{2};b,b+\frac{1}{2};t^2) = \frac{\Gamma(2b)}{\Gamma(2a_2)} A(2a_2-1)\tilde{A}(2b_2-1)(1-t^2)^{-a_1}$

66. $_3F_2(-n,n+1,a;1,b;t) = \frac{\Gamma(b)}{\Gamma(a)} A(a-1)\tilde{A}(b-1)P_n(1-2t)$

67. $_3F_2(-n,n+a_1,a_2;\frac{a_1+1}{2},b;t) = \frac{\Gamma(b)n!\Gamma(a_1)}{\Gamma(a_1+n)\Gamma(a_2)} A(a_2-1)\tilde{A}(b-1)C_n^{(a_1)/2}(1-2t)$

68. $_3F_2(\alpha+n+1,-\beta-n,a;\alpha+1,b;\pm t) = \frac{n!\Gamma(\alpha+1)\Gamma(b)}{\Gamma(\alpha+n+1)\Gamma(a)} A(a-1)\tilde{A}(b-1)(1\mp t)^{\beta}P_n^{\alpha,\beta}(1\mp 2t)$

69. $_3F_2(-\nu,1+\nu,\kappa;1-\lambda,\kappa+\mu;t)H(1-t)$

$$= \frac{\Gamma(\kappa+\mu)\Gamma(1-\lambda)}{\Gamma(\kappa)} A(\kappa-1)\tilde{A}(\mu+\kappa-1)t^{\lambda/2}\cdot(1-t)^{-\lambda/2}P_\nu^\lambda(1-2t)$$

70. $${}_3F_2(a_1,a_2,a_3;b_1,b_2;t) = \Gamma(b_2)\tilde{A}(b_2-1)\tilde{M}_3F_1(a_1,a_2,a_3;b_1;t)$$

71. $${}_3F_2(a_1,a_2,a_3;b_1,b_2;t) = \frac{1}{\Gamma(b_3)} A(b_3-1)M_3F_3(a_1,a_2,a_3;b_1,b_2,b_3;t)$$

72. $${}_3F_2(a_1,a_2,a_3;b_1,b_2;t) = \frac{1}{\Gamma(a_3)} A(a_3-1)M_2F_2(a_1,a_2;b_1,b_2;t)$$

73. $${}_3F_2(a_1,a_2,a_3;b_1,b_2;t) = \Gamma(a_4)\tilde{A}(a_4-1)\tilde{M}_4F_2(a_1,a_2,a_3,a_4;b_1,b_2;t)$$

74. $${}_3F_2(a_1,a_2,a_3;b_1,b_2;-t^{-1}) = \frac{\Gamma(b_1)\Gamma(b_2)}{\Gamma(a_1)\Gamma(a_2)\Gamma(a_3)} B(a_1)\tilde{B}(b_1)B(a_2)\tilde{B}(b_2)B(a_3)Ne^{-1/t}$$

75. $${}_3F_2(a_1,a_2,a_2+\tfrac{1}{2};b,b+\tfrac{1}{2};t^{-2}) = \frac{\Gamma(2b)}{\Gamma(2a_2)} B(2a_2)\tilde{B}(2b_2)t^{-2a_1}(t^2-1)^{-a_1}$$

76. $${}_3F_2(-n,n+1,a;1,b;t^{-1}) = \frac{\Gamma(b)}{\Gamma(a)} B(a)\tilde{B}(b)P_n(1-\tfrac{2}{t})$$

77. $${}_3F_2(-n,n+a_1,a_2;\frac{a_1+1}{2},b;t^{-1}) = \frac{\Gamma(b)n!\Gamma(a_1)}{\Gamma(a_1+n)\Gamma(a_2)} B(a_2)\tilde{B}(b)C_n^{\frac{1}{2}a_1}(1-\tfrac{2}{t})$$

78. $${}_3F_2(a_1,a_2,a_3;b_1,b_2;t^{-1}) = \Gamma(b_2)\tilde{B}(b_2)\tilde{M}_3F_1(a_1,a_2,a_3;b_1;t^{-1})$$

79. $${}_3F_2(a_1,a_2,a_3;b_1,b_2;t^{-1}) = \frac{1}{\Gamma(b_3)} B(b_3)M_3F_3(a_1,a_2,a_3;b_1,b_2,b_3;t^{-1})$$

80. $${}_3F_2(a_1,a_2,a_3;b_1,b_2;t^{-1}) = \frac{1}{\Gamma(a_3)} B(a_3)M_2F_2(a_1,a_2;b_1,b_2;t^{-1})$$

81. $${}_3F_2(a_1,a_2,a_3;b_1,b_2;t^{-1}) = \Gamma(a_4)\tilde{B}(a_4)\tilde{M}_4F_2(a_1,a_2,a_3,a_4;b_1,b_2;t^{-1})$$

82. $${}_mF_{n+1}(a_1,\ldots,a_m;b_1,\ldots,b_{n+1};t) = \Gamma(b_{n+1})\widetilde{A}(b_{n+1}-1)\widetilde{M}_m F_n(a_1,\ldots,a_m;b_1,\ldots,b_n;t)$$

83. $${}_mF_n(a_1,\ldots,a_m;b_1,\ldots,b_n;t) = \frac{1}{\Gamma(b_{n+1})} A(b_{n+1}-1)M_m F_{n+1}(a_1,\ldots,a_m;b_1,\ldots,b_{n+1};t)$$

84. $${}_{m+1}F_n(a_1,\ldots,a_{m+1};b_1,\ldots,b_n;t) = \frac{1}{\Gamma(a_{m+1})} A(a_{m+1}-1)M_m F_n(a_1,\ldots,a_m;b_1,\ldots,b_n;t)$$

85. $${}_mF_n(a_1,\ldots,a_m;b_1,\ldots,b_n;t) = \Gamma(a_{m+1})\widetilde{A}(a_{m+1}-1)\widetilde{M}_{m+1} F_n(a_1,\ldots,a_{m+1};b_1,\ldots,b_n;t)$$

86. $${}_{n+1}F_n\left(a,\frac{b}{n},\frac{b+1}{n},\ldots,\frac{b+n-1}{n};\frac{c}{n},\frac{c+1}{n},\ldots,\frac{c+n-1}{n};t^n\right) = \frac{\Gamma(c)}{\Gamma(b)} A(b-1)\widetilde{A}(c-1)(1-t^n)^{-a}$$

87. $${}_nF_n\left(\frac{b}{n},\frac{b+1}{n},\ldots,\frac{b+n-1}{n};\frac{c}{n},\ldots,\frac{c+n-1}{n};t^n\right) = \frac{\Gamma(c)}{\Gamma(b)} A(b-1)\widetilde{A}(c-1)e^{t^n}$$

88. $${}_mF_n(a_1,\ldots,a_m;b_1,\ldots,b_n;-t^{-1/2})$$

$$= \frac{1}{\sqrt{\pi}} A\left(-\frac{1}{2}\right)M_{2m}F_{2n}\left(\frac{a_1}{2},\frac{a_1+1}{2},\ldots,\frac{a_m}{2},\frac{a_{m+1}}{2};\frac{b_1}{2},\frac{b_1+1}{2},\ldots,\frac{b_n}{2},\frac{b_{n+1}}{2};2^{-m-n-2}t^{-1}\right)$$

89. $${}_mF_{n+1}(a_1,\ldots,a_m;b_1,\ldots,b_{n+1};t^{-1}) = \Gamma(b_{n+1})\widetilde{B}(b_{n+1})\widetilde{N}_m F_n(a_1,\ldots,a_m;b_1,\ldots,b_n;t^{-1})$$

90. $${}_mF_n(a_1,\ldots,a_m;b_1,\ldots,b_n;t^{-1}) = \frac{1}{\Gamma(b_{n+1})} B(b_{n+1})N_m F_{n+1}(a_1,\ldots,a_m;b_1,\ldots,b_{n+1};t^{-1})$$

91. $${}_{m+1}F_n(a_1,\ldots,a_{m+1};b_1,\ldots,b_n;t^{-1}) = \frac{1}{\Gamma(a_{m+1})} B(a_{m+1})N\,\cdot$$

$$\cdot\ {}_mF_n(a_1,\ldots,a_m;b_1,\ldots,b_n;t^{-1})$$

92. $\quad {}_mF_n(a_1,\ldots,a_m;b_1,\ldots,b_n;t^{-1}) = \Gamma(a_{m+1})\tilde{B}(a_{m+1})\tilde{N}_{m+1}F_n(a_1,\ldots,a_{m+1};b_1,\ldots,b_n;t^{-1})$

93. $\quad {}_{n+1}F_n(a,\frac{b}{n},\frac{b+1}{n},\ldots,\frac{b+n-1}{n};\frac{c}{n},\frac{c+1}{n},\ldots,\frac{c+n-1}{n};t^{-n}) = \frac{\Gamma(c)}{\Gamma(b)}\,B(b)\tilde{B}(c)t^{na}(t^n-1)^{-a}$

94. $\quad {}_nF_n(\frac{b}{n},\frac{b+1}{n},\ldots,\frac{b+n-1}{n};\frac{c}{n},\ldots,\frac{c+n-1}{n};t^{-n}) = \frac{\Gamma(c)}{\Gamma(b)}\,B(b)\tilde{B}(c)e^{t^{-n}}$

95. $\quad {}_mF_n(a_1,\ldots,a_m;b_1,\ldots,b_n;-t^{\frac{1}{2}})$

$$= \frac{1}{\sqrt{\pi}}\,B(\tfrac{1}{2})N_{2m}F_{2n}(\frac{a_1}{2},\frac{a_1+1}{2},\ldots,\frac{a_m}{2},\frac{a_m+1}{2};\frac{b_1}{2},\frac{b_1+1}{2},\ldots,\frac{b_n}{2},\frac{b_n+1}{2};2^{-m-n-2}t)$$

96. $\quad {}_mF_n(a_1,\ldots,a_m;\mu+\nu,b_2,\ldots,b_n;t) = \frac{\Gamma(\mu+\nu)}{\Gamma(\nu)}\,A(\nu-1)A(\nu+\mu-1)\,\cdot$

$$\cdot\ {}_mF_n(a_1,\ldots,a_m;\nu,b_2,\ldots,b_n;t)$$

97. $\quad {}_{m+1}F_{n+1}(\nu,a_1,\ldots,a_m;\mu+\nu,b_1,\ldots,b_n;t) = \frac{\Gamma(\mu+\nu)}{\Gamma(\nu)}\,A(\nu-1)A(\nu+\mu-1)\,\cdot$

$$\cdot\ {}_mF_n(a_1,\ldots,a_m;b_1,\ldots,b_n;t)$$

98. $\quad {}_mF_n(a_1,\ldots,a_m;\mu+\nu,b_2,\ldots,b_n;t^{-1}) = \frac{\Gamma(\mu+\nu)}{\Gamma(\nu)}\,B(\nu)B(\nu+\mu)\,\cdot$

$$\cdot\ {}_mF_n(a_1,\ldots,a_m;\nu,b_2,\ldots,b_n;t^{-1})$$

99. $${}_{m+1}F_{n+1}(\nu,a_1,\ldots,a_m;\mu+\nu,b_1,\ldots,b_n;t) = \frac{\Gamma(\mu+\nu)}{\Gamma(\nu)} B(\nu)\widetilde{B}(\nu+\mu) \cdot$$

$$\cdot\ {}_mF_n(a_1,\ldots,a_m;b_1,\ldots,b_n;t^{-1})$$

100. $$E(\alpha,\beta,\gamma:\delta:t) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\delta)} B(\gamma)NF(\alpha,\beta;\delta;-t^{-1})$$

101. $$E(\alpha,\beta,\gamma:\delta:t^{-1}) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\delta)} A(\gamma-1)MF(\alpha,\beta;\delta;-t)$$

102. $$E(\delta-\alpha,\delta-\beta,\gamma:\delta:t) = \frac{\Gamma(\delta-\alpha)\Gamma(\delta-\beta)}{\Gamma(\delta)} B(\gamma)Nt^{\gamma-\alpha-\beta}(t+1)^{\alpha+\beta-\gamma}F(\alpha,\beta;\delta;-t^{-1})$$

103. $$E(\delta-\alpha,\delta-\beta,\gamma:\delta:t^{-1}) = \frac{\Gamma(\delta-\alpha)\Gamma(\delta-\beta)}{\Gamma(\delta)} A(\gamma-1)M(1+t)^{\alpha+\beta-\gamma}F(\alpha,\beta;\delta;-t)$$

<u>Confluent hypergeometric functions</u>

1. $\Phi(a,c;t) = {}_1F_1(a;c;t) = \frac{\Gamma(c)}{\Gamma(a)} A(a-1)\tilde{A}(c-1)e^t$

2. $\Phi(a,c;t) = \frac{\Gamma(c)}{\Gamma(c-a)} e^t A(c-a-1)\tilde{A}(c-1)e^{-t}$

3. $\Phi(a,c;t) = \Gamma(c)\tilde{A}(c-1)\tilde{M}(1-t)^{-a}$

4. $\Phi(a,c;t) = \Gamma(c)\tilde{A}(c-1)\tilde{M}_1F_0(a;;t)$

5. $\Phi(a,c;t) = \frac{1}{\Gamma(\sigma)} A(\sigma-1)M_1F_2(a;c,\sigma;t)$

6. $\Phi(a,c;t) = \frac{1}{\Gamma(a)} A(a-1)M_0F_1(c;t)$

7. $\Phi(a,c;t) = \Gamma(b)\tilde{A}(b-1)\tilde{M}_2F_1(a,b;c;t)$

8. $\Phi(\nu + \frac{1}{2},c+2\nu+1;t) = \frac{\sqrt{\pi}\ \Gamma(c+2\nu+1)}{\Gamma(\nu+\frac{1}{2})} A(2\nu)\tilde{A}(c+2\nu)t^{-\nu}e^{t/2}I_\nu(\frac{t}{2})$

9. $\Phi(\nu + \frac{1}{2},c+2\nu+1;\pm it) = \frac{\sqrt{\pi}\ \Gamma(c+2\nu+1)}{\Gamma(\nu+\frac{1}{2})} A(2\nu)\tilde{A}(c+2\nu)t^{-\nu}e^{\pm it/2}J_\nu(\frac{t}{2})$

10. $\Phi(a,c;t^{-1}) = \frac{\Gamma(c)}{\Gamma(a)} B(a)\tilde{B}(c)e^{1/t}$

11. $\Phi(a,c;t^{-1}) = e^{1/t}B(c-a)\tilde{B}(c)e^{-1/t}$

12. $\phi(a,c;t^{-1}) = \Gamma(c)\tilde{B}(c)\tilde{N}t^{a}(t-1)^{-a}$

13. $\phi(a,c;t^{-1}) = \Gamma(c)\tilde{B}(c)\tilde{N}\,{}_1F_0(a;;t^{-1})$

14. $\phi(a,c;t^{-1}) = \frac{1}{\Gamma(\sigma)} B(\sigma)N\,{}_1F_2(a;c,\sigma;t^{-1})$

15. $\phi(a,c;t^{-1}) = \frac{1}{\Gamma(a)} B(a)N\,{}_0F_1(c;t^{-1})$

16. $\phi(a,c;t^{-1}) = \Gamma(b)\tilde{B}(b)\tilde{N}\,{}_2F_1(a,b;c;t^{-1})$

17. $\phi(\nu + \frac{1}{2},c;t^{-1}) = \frac{\sqrt{\pi}\;\Gamma(c)}{\Gamma(\nu+\frac{1}{2})} B(2\nu+1)\tilde{B}(c)t^{\nu}e^{2/t}I_{\nu}(\frac{1}{2t})$

18. $.(t-1)^{c-1}\phi(a,c;t-1) = \frac{\Gamma(c)}{\Gamma(a)} A(a-c)t^{c-a}.(t-1)^{a-1}e^{t-1}$

19. $.(t-1)^{c-1}\phi(a,c;1-t) = \frac{\Gamma(c)}{\Gamma(c-a)} e^{-t}A(-a)t^{a}e^{t}.(t-1)^{a-1}$

20. $\psi(a,c;t) = e^{t}\tilde{B}(a-c+1)t^{1-c}e^{-t}$

21. $\psi(a,c;t) = \frac{1}{\Gamma(a)} Nt^{1-c}(1+t)^{c-a-1}$

22. $\psi(a,c;t) = e^{t}B(1-c)\tilde{B}(a-c+1)e^{-t}$

23. $\psi(a,c;t) = \frac{1}{\Gamma(a)\Gamma(1+a-c)}NNB(1-c)t^{-a}e^{-1/t}$

24. $\psi(a,c;t) = \frac{1}{\Gamma(a)\Gamma(1+a-c)} NMA(a-1)t^{1-c}e^{-t}$

25. $\psi(a,c;t) = \frac{1}{\Gamma(a)} e^{t}Nt^{1-c}.(1-t)^{a-1}$

26. $\psi(a,c;t) = \frac{e^{t}}{\Gamma(a-c+1)} B(1-c)N.(1-t)^{a-c}$

27. $\psi(a,c;t^{-1}) = e^{1/t}A(-1)\tilde{A}(a-c)t^{c-1}e^{-1/t}$

28. $\psi(a,c;t^{-1}) = e^{1/t}A(-c)\tilde{A}(a-c)e^{-1/t}$

29. $\psi(a,c;t^{-1}) = \frac{1}{\Gamma(a)} e^{1/t}A(-1)Mt^{c-a}.(t-1)^{a-1}$

30. $\psi(a,c;t^{-1}) = \frac{e^{1/t}}{\Gamma(a-c+1)} A(-c)Mt^{a-c}.(t-1)^{a-c}$

31. $\psi(a,c;t-1) = e^{t}\tilde{B}(a-c+1)t^{a-c+1}e^{-t}(t-1)^{-a}$

32. $\psi(a,c;t-1) = e^{t}(t-1)^{1-c}\tilde{B}(a)t^{a}(t-1)^{c-a-1}e^{-t}$

33. $W_{\kappa+\frac{1}{2}\mu,\lambda+\frac{1}{2}\mu}(t) = \frac{\Gamma(\frac{1}{2}-\kappa-\lambda)e^{-\frac{1}{2}t}}{\Gamma(\frac{1}{2}-\kappa-\lambda-\mu)} B(\frac{1-\mu}{2} - \lambda)\tilde{B}(\frac{1+\mu}{2} - \lambda)t^{\mu/2}e^{\frac{1}{2}t}W_{\kappa,\mu}(t)$

34. $W_{\kappa-\mu,\lambda}(t) = e^{\frac{1}{2}t}B(1-\kappa)\tilde{B}(1+\mu-\kappa)e^{-\frac{1}{2}t}W_{\kappa,\lambda}(t)$

35. $W_{\kappa-\frac{1}{2}\mu,\lambda-\frac{1}{2}\mu}(t) = e^{\frac{1}{2}t}B(\frac{1-\mu}{2} - \lambda)\tilde{B}(\frac{1+\mu}{2} - \lambda)t^{\mu/2}e^{-\frac{1}{2}t}W_{\kappa,\lambda}(t)$

36. $W_{\kappa+\frac{1}{2}\mu,\lambda+\frac{1}{2}\mu}(t^{-1}) = \frac{\Gamma(\frac{1}{2}-\kappa-\lambda)e^{-\frac{1}{2}t}}{\Gamma(\frac{1}{2}-\kappa-\lambda-\mu)} A(-\lambda - \frac{\mu+1}{2})\tilde{A}(\frac{\mu-1}{2} - \lambda)t^{-\mu/2}e^{\frac{1}{2}t}W_{\kappa,\lambda}(t^{-1})$

37. $W_{\kappa-\mu,\lambda}(t^{-1}) = e^{\frac{1}{2}t}A(-\kappa)\tilde{A}(\mu-\kappa)e^{-\frac{1}{2}t}W_{\kappa,\lambda}(t^{-1})$

38. $W_{\kappa-\frac{1}{2}\mu,\lambda-\frac{1}{2}\mu}(t^{-1}) = e^{\frac{1}{2}t}A(-\frac{\mu+1}{2} - \lambda)\tilde{A}(\frac{\mu-1}{2} - \lambda)t^{-\mu/2}e^{-\frac{1}{2}t}W_{\kappa,\lambda}(t^{-1})$

39. $W_{\alpha,\gamma}(t^{-1}) = e^{\frac{1}{2}t}A(\gamma - \frac{1}{2})\tilde{A}(-\alpha)t^{\gamma-\frac{1}{2}}e^{-1/t}$

40. $W_{\alpha,\gamma}(t^{-1}) = e^{\frac{1}{2}t}A(-\gamma - \frac{1}{2})\tilde{A}(-\alpha)t^{-\gamma-\frac{1}{2}}e^{-1/t}$

41. $W_{\kappa,\mu}(t^{-1}) = -e^{\frac{1}{2}t}B(\kappa)Nt^{-\frac{1}{2}}\{J_{2\mu}(2t^{-\frac{1}{2}})\sin(\mu-\kappa)\pi + Y_{2\mu}(2t^{-\frac{1}{2}})\cos(\mu-\kappa)\pi\}$

42. $W_{\kappa,\mu}(t^{-1}) = \frac{2}{\Gamma(\frac{1}{2}-\kappa+\mu)\Gamma(\frac{1}{2}-\kappa-\mu)} B(-\kappa)Nt^{-\frac{1}{2}}K_{2\mu}(2t^{-\frac{1}{2}})$

43. $W_{\kappa,\mu}(t) = \frac{2}{\Gamma(\frac{1}{2}-\kappa+\mu)\Gamma(\frac{1}{2}-\kappa-\mu)} A(-\kappa-1)Mt^{\frac{1}{2}}K_{2\mu}(2t^{\frac{1}{2}})$

44. $W_{\kappa,\mu}(t) = -e^{t/2}MA(\kappa-1)t^{\frac{1}{2}}\{J_{2\mu}(2t^{\frac{1}{2}})\sin(\mu-\kappa)\pi + Y_{2\mu}(2t^{\frac{1}{2}})\cos(\mu-\kappa)\pi\}$

45. $W_{\kappa,\mu}(t) = e^{t/2}B(\mu + \frac{1}{2})\tilde{B}(1-\kappa)t^{\frac{1}{2}-\mu}e^{-t}$

46. $W_{\kappa,\mu}(t) = e^{t/2}B(\frac{1}{2} - \mu)\tilde{B}(1-\kappa)t^{\mu+\frac{1}{2}}e^{-t}$

47. $M_{\kappa,\mu}(t) = \frac{\Gamma(2\mu+1)}{\Gamma(\mu-\kappa+\frac{1}{2})} e^{-t/2}A(-\kappa-1)Mt^{\frac{1}{2}}I_{2\mu}(2t^{\frac{1}{2}})$

48. $M_{\kappa,\mu}(t) = \frac{\Gamma(2\mu+1)}{\Gamma(\mu+\frac{1}{2}-<)} e^{-t/2}A(-\kappa-1)\widetilde{A}(\mu - \frac{1}{2})t^{\mu+\frac{1}{2}}e^{t}$

49. $M_{\kappa,\mu}(t) = \frac{\Gamma(2\mu+1)}{\Gamma(\mu+\frac{1}{2}-\kappa)} e^{t/2}A(\kappa-1)\widetilde{A}(\mu - \frac{1}{2})t^{\mu+\frac{1}{2}}e^{-t}$

50. $M_{\kappa,\mu}(t^{-1}) = \frac{\Gamma(2\mu+1)}{\Gamma(\mu-\kappa+\frac{1}{2})} e^{-\frac{1}{2}t}B(-\kappa)\widetilde{B}(\mu + \frac{1}{2})t^{-\mu-\frac{1}{2}}e^{1/t}$

51. $M_{\kappa,\mu}(t^{-1}) = \frac{\Gamma(2\mu+1)}{\Gamma(\mu+\frac{1}{2}-\kappa)} e^{-\frac{1}{2}t}B(-\kappa)Nt^{-\frac{1}{2}}I_{2\mu}(2t^{-\frac{1}{2}})$

52. $M_{\kappa,\mu}(t^{-1}) = \frac{\Gamma(2\mu+1)}{\Gamma(\mu-\kappa+2)} e^{\frac{1}{2}t}B(\kappa)\widetilde{B}(\mu + \frac{1}{2})t^{-\mu-\frac{1}{2}}e^{-1/t}$

53. $D_{\nu}(t^{\frac{1}{2}}) = 2^{\nu/2}e^{t/4}\widetilde{B}(\frac{1-\nu}{2})t^{\frac{1}{2}}e^{-t/2}$

54. $D_{-2\nu}(t^{\frac{1}{2}}) = \frac{2^{\nu-1}}{\Gamma(2\nu)} e^{-t/4}A(\nu-1)Me^{-\sqrt{2}}\ t^{\frac{1}{2}}$

55. $D_{\nu}(t^{-\frac{1}{2}}) = 2^{\nu/2}e^{1/4t}A(-1)\widetilde{A}(-\ \frac{\nu+1}{2})t^{-\frac{1}{2}}e^{-\frac{1}{2}t}$

56. $D_{-2\nu}(t^{\frac{1}{2}}) = \frac{2^{\nu-1}}{\Gamma(2\nu)} e^{-1/4t}B(\nu)Ne^{-\sqrt{2}}\ /t^{\frac{1}{2}}$

List of Abbreviations, Symbols and Notations

$\binom{a}{b}$ = Binomial coefficient, $\binom{a}{b} = \frac{\Gamma(a+1)}{\Gamma(b+1)\Gamma(a-b+1)}$

$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}$

$.(1-t)^{\alpha} = (1-t)^{\alpha}H(1-t)$

$.(t-1)^{\alpha} = (t-1)^{\alpha}H(t-1)$

1. Elementary functions

Trigonometric and inverse trigonometric functions:

$\sin x$, $\cos x$, $\tan x = \sin x/\cos x$, $\mathrm{ctn}\, x = \cos x/\sin x$,

$\sec x = 1/\cos x$, $\csc x = 1/\sin x$, $\arcsin x$, $\arccos x$, $\arctan x$,

$\mathrm{arcctn}\, x$

Hyperbolic functions:

$\sinh x = (e^x - e^{-x})/2$, $\cosh x = (e^x + e^{-x})/2$, $\tanh x = \sinh x/\cosh x$,

$\mathrm{ctnh}\, x = \cosh x/\sinh x$, $\mathrm{sech}\, x = 1/\cosh x$, $\mathrm{csch}\, x = 1/\sinh x$.

2. Orthogonal polynomials

Legendre polynomials:

$$P_n(x) = 2^{-n}(n!)^{-1} \frac{d^n}{dx^n} (x^2-1)^n = {}_2F_1(-n,n+1;1;\tfrac{1-x}{2})$$

Gegenbauer's polynomials:

$$C_n^{\alpha}(x) = [n!\Gamma(2\alpha)]^{-1}\Gamma(2\alpha+n)\,_2F_1(-n,2\alpha+n;\alpha+1/2;\ \tfrac{1-x}{2})$$

Chebychev polynomials:

$$T_n(x) = \cos(n \arccos x) = \,_2F_1(-n,n;\tfrac{1}{2};\ \tfrac{1-x}{2}) = \frac{n}{2} \lim_{\alpha=0} \Gamma(\alpha)C_n^{\alpha}(x)$$

$$U_n(x) = (1-x^2)^{-\frac{1}{2}}\sin[(n+1)\arccos x]$$

$$= x(n+1)\,_2F_1(\tfrac{1-n}{2},\ \tfrac{3+n}{2};\tfrac{3}{2};1-x^2)$$

Jacobi polynomials:

$$P_n^{(\beta,\alpha)}(x) = [n!\Gamma(1+\beta)]^{-1}\Gamma(1+\beta+n)\,_2F_1(-n,n+\alpha+\beta+1;\beta+1;\ \tfrac{1-x}{2})$$

Laguerre polynomials:

$$L_n^{\alpha}(x) = (n!)^{-1}x^{-\alpha}e^{x}\frac{d^n}{dx^n}(e^{-x}x^{n+\alpha}) = [n!\Gamma(1+\alpha)]^{-1}\Gamma(\alpha+1+n)\,_1F_1(-n;1+\alpha;x)$$

$$L_n(x) = L_n^0(x)$$

Hermite polynomials:

$$He_n(x) = (-1)^n \exp(x^2/2) \frac{d^n}{dx^n} \exp(-x^2/2)$$

$$He_{2n}(x) = (-1)^n 2^{-n} (n!)^{-1} (2n)!\, {}_1F_1(-n; \tfrac{1}{2}; \tfrac{1}{2} x^2)$$

$$He_{2n+1}(x) = (-1)^n 2^{-n} (n!)^{-1} (2n+1)!\, x\, {}_1F_1(-n; \tfrac{3}{2}; \tfrac{1}{2} x^2)$$

3. Gamma function and related functions

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt, \qquad \mathrm{Re}\, z > 0$$

Beta function:

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

4. Legendre functions (definition according to Hobson)

$$P_\alpha^\beta(z) = [\Gamma(1-\beta)]^{-1} \left(\frac{z+1}{z-1}\right)^{\beta/2} {}_2F_1(-\alpha, \alpha+1; 1-\beta; \frac{1-z}{2})$$

$$Q_\alpha^\beta(z) = 2^{-\alpha-1} [\Gamma(\alpha+3/2)]^{-1} e^{i\pi\beta} \sqrt{\pi}\, \Gamma(\alpha+\beta+1) z^{-\alpha-\beta-1} (z^2-1)^{\beta/2} \cdot$$

$$\cdot\ {}_2F_1\left(\frac{\alpha+\beta+1}{2}, \frac{\alpha+\beta+2}{2}; \alpha + \frac{3}{2}; z^{-2}\right)$$

z is a point in the complex z plane cut along the real axis from $-\infty$ to $+1$.

$$P_\alpha^\beta(x) = [\Gamma(1-\beta)]^{-1}\left(\frac{1+x}{1-x}\right)^{\beta/2} {}_2F_1(-\alpha,\alpha+1;1-\beta;\frac{1-x}{2}), \qquad -1 < x < 1$$

$$Q_\alpha^\beta(x) = \frac{1}{2}\, e^{-i\pi\beta}[e^{-i\pi\beta/2}Q_\alpha^\beta(x+i0) + e^{i\pi\beta/2}Q_\alpha^\beta(x-i0)], \qquad -1 < x < 1$$

$$P_\alpha(z) = P_\alpha^0(z);\quad Q_\alpha(z) = Q_\alpha^0(z);\quad P_\alpha(x) = P_\alpha^0(x);\quad Q_\alpha(x) = Q_\alpha^0(x)$$

5. Bessel functions

$$J_\nu(z) = \sum_{n=0}^{\infty} \frac{(-1)^n (z/2)^{\nu+2n}}{n!\,\Gamma(\nu+n+1)}$$

$$Y_\nu(z) = \mathrm{ctn}(\pi\nu)J_\nu(z) - \csc(\pi\nu)J_{-\nu}(z)$$

$$H_\nu^{(1)}(z) = J_\nu(z) + iY_\nu(z);\quad H_\nu^{(2)}(z) = J_\nu(z) - iY_\nu(z)$$

6. Modified Bessel functions

$$I_\nu(z) = e^{-i\pi\nu/2}J_\nu(ze^{i\pi/2}) = \sum_{n=0}^{\infty} \frac{(z/2)^{\nu+2n}}{n!\,\Gamma(\nu+n+1)}$$

$$K_\nu(z) = \frac{1}{2}\,\pi\,\csc(\pi\nu)[I_{-\nu}(z)-I_\nu(z)]$$

$$= \frac{1}{2}\, i\,\pi\, e^{i\pi\nu/2}H_\nu^{(1)}(ze^{i\pi/2}) = -\frac{1}{2}\, i\,\pi\, e^{-i\pi\nu/2}H_\nu^{(2)}(ze^{-i\pi/2})$$

7. Struve functions

$$\mathbf{H}_\nu(z) = \sum_{n=0}^{\infty} \frac{(-1)^n (z/2)^{\nu+2n+1}}{\Gamma(n+3/2)\Gamma(\nu+n+3/2)} = 2^{1-\nu}\pi^{-\frac{1}{2}}[\Gamma(\nu + \tfrac{1}{2})]^{-1} s_{\nu,\nu}(z)$$

$$\mathbf{L}_\nu(z) = -ie^{-\pi\nu/2}\mathbf{H}_\nu(ze^{i\pi/2})$$

8. Lommel functions

$$s_{\alpha,\beta}(z) = [(\alpha-\beta+1)(\alpha+\beta+1)]^{-1} z^{\alpha+1} {}_1F_2(1;\tfrac{\alpha-\beta+3}{2}, \tfrac{\alpha+\beta+3}{2};-z^2/4); \quad \alpha \pm \beta \neq -1,-2,-3,\ldots$$

$$S_{\alpha,\beta}(z) = s_{\alpha,\beta}(z) + 2^{\alpha-1}\Gamma(\tfrac{\alpha-\beta+1}{2})\Gamma(\tfrac{\alpha+\beta+1}{2})[\sin(\tfrac{\pi\alpha-\pi\beta}{2})J_\alpha(z)-\cos(\tfrac{\pi\alpha-\pi\beta}{2})Y_\alpha(z)].$$

Special cases of Lommel's functions:

$$s_{\alpha,\alpha}(z) = \pi^{\frac{1}{2}}2^{\alpha-1}\Gamma(\alpha + \tfrac{1}{2})\mathbf{H}_\alpha(z)$$

$$S_{\alpha,\alpha}(z) = \pi^{\frac{1}{2}}2^{\alpha-1}\Gamma(\alpha + \tfrac{1}{2})[\mathbf{H}_\alpha(z) - Y_\alpha(z)]$$

$$s_{0,\beta}(z) = \frac{1}{2}\pi \csc(\pi\beta)[\mathbf{J}_\beta(z) - \mathbf{J}_{-\beta}(z)]$$

$$S_{0,\beta}(z) = \frac{\pi}{2}\csc(\pi\beta)[\mathbf{J}_\beta(z) - \mathbf{J}_{-\beta}(z) - J_\beta(z) + J_{-\beta}(z)]$$

$$s_{-1,\beta}(z) = -\frac{\pi}{2}\beta^{-1}\csc(\pi\beta)[\mathbf{J}_\beta(z) + \mathbf{J}_{-\beta}(z)]$$

$$S_{-1,\beta}(z) = \frac{\pi}{2}\beta^{-1}\csc(\pi\beta)[J_\beta(z) + J_{-\beta}(z) - \mathbf{J}_\beta(z) - \mathbf{J}_{-\beta}(z)]$$

$$s_{1,\beta}(z) = 1 + \beta^2 s_{-1,\beta}(z); \quad S_{1,\beta}(z) = 1 + \beta^2 S_{-1,\beta}(z)$$

$$S_{1/2,1/2}(z) = z^{-\frac{1}{2}}; S_{3/2,1/2}(z) = z^{\frac{1}{2}}$$

$$S_{-1/2,1/2}(z) = z^{-\frac{1}{2}}[\sin z\ Ci(z) - \cos z\ si(z)];$$

$$S_{-3/2,1/2}(z) = -z^{-\frac{1}{2}}[\sin z\ si(z) + \cos z\ Ci(z)]$$

$$\lim_{\alpha\to\beta} [\Gamma(\beta-\alpha)]^{-1} s_{\alpha-1,\beta}(z) = -2^{\beta-1}\Gamma(\beta)J_\beta(z)$$

9. Gauss's hypergeometric function

$$_2F_1(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{z^n}{n!}, \quad |z| < 1$$

10. Generalized hypergeometric series

$$_mF_n(a_1,a_2,\ldots,a_m;b_1,b_2,\ldots,b_n;z) = \frac{\Gamma(b_1)\cdots\Gamma(b_n)}{\Gamma(a_1)\cdots\Gamma(a_m)} \sum_{k=0}^{\infty} \frac{\Gamma(a_1+k)\cdots\Gamma(a_m+k)}{\Gamma(b_1+k)\cdots\Gamma(b_n+k)} \frac{z^k}{k!}$$

11. Confluent hypergeometric functions

$$_1F_1(a;c;z) = \frac{\Gamma(c)}{\Gamma(a)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)}{\Gamma(c+n)} \frac{z^n}{n!}$$

$$_1F_1(a;a;z) = e^z, \quad _1F_1(a;2a;2z) = 2^{a-\frac{1}{2}}\Gamma(a+\tfrac{1}{2})z^{\frac{1}{2}-a}e^z I_{a-\frac{1}{2}}(z)$$

$$_1F_1(\tfrac{1}{2};\tfrac{3}{2};ix) = e^{ix}\,_1F_1(1;\tfrac{3}{2};-ix) = (\tfrac{1}{2}\pi/x)^{\frac{1}{2}}[C(x) + i\,S(x)]$$

Whittaker's functions:

$$M_{\alpha,\mu}(z) = z^{\mu+\frac{1}{2}}e^{-\frac{1}{2}z}\,{}_1F_1(\mu-\alpha+\frac{1}{2};2\mu+1:z)$$

$$W_{\alpha,\mu}(z) = \frac{\Gamma(-2\mu)}{\Gamma(-\alpha-\mu+1/2)}M_{\alpha,\mu}(z) + \frac{\Gamma(2\mu)}{\Gamma(\mu-\alpha+1/2)}M_{\alpha,-\mu}(z)$$

Special cases of Whittaker's functions:

$$M_{0,\mu}(z) = \Gamma(1+\mu)2^{2\mu}I_{\mu}(z/2)\sqrt{z}\ ;\quad W_{0,\mu}(z) = (z/\pi)^{\frac{1}{2}}K_{\mu}(z/2)$$

$$M_{\alpha,0}(z) = z^{\frac{1}{2}}e^{-\frac{1}{2}z}L_{\alpha-\frac{1}{2}}(z);\ M_{1/4,1/4}(z) = -i\,\frac{1}{2}\,\pi^{\frac{1}{2}}z^{\frac{1}{4}}e^{-\frac{1}{2}z}\mathrm{Erf}(iz^{\frac{1}{2}})$$

Parabolic cylinder function:

$$D_{\alpha}(z) = 2^{(\alpha+\frac{1}{2})/2}z^{-\frac{1}{2}}W_{(\alpha+\frac{1}{2})/2,\frac{1}{4}}(z^2/2)$$

$$D_n(z) = e^{-z^2/4}He_n(z),\quad n=0,1,2,\ldots$$

$$D_{-1}(z) = (\pi/2)^{\frac{1}{2}}e^{z^2/4}\mathrm{Erfc}(2^{-\frac{1}{2}}z)$$

$$D_{-\frac{1}{2}}(z) = (\frac{1}{2}\,z/\pi)^{\frac{1}{2}}K_{\frac{1}{4}}(z^2/4)$$

Symbol	Name of the Function	Listed under
$C_n^{\alpha}(x)$	Gegenbauer's polynomial	2
$D_{\alpha}(x)$	Parabolic cylinder function	11
${}_mF_n$	Hypergeometric function	9,10,11
$H_{\alpha}^{(1,2)}(x)$	Hankel's functions	5
$\mathbf{H}_{\alpha}(z)$	Struve's function	7
$I_{\nu}(z)$	Modified Bessel function	6
$J_{\nu}(z)$	Bessel's function	5
$\mathbf{J}_{\nu}(z)$	Anger-Weber function	8
$K_{\nu}(z)$	Modified Hankel function	6
$L_n^{\alpha}(x)$	Laguerre's polynomial	2
$\mathbf{L}_{\nu}(z)$	Struve's function	7
$M_{\alpha,\beta}(z)$, $W_{\alpha,\beta}(z)$	Whittaker's functions	11
$P_n(x)$	Legendre's polynomials	2
$P_n^{(\alpha,\beta)}(x)$	Jacobi's polynomials	2
$P_{\alpha}^{\beta}(z)$, $P_{\alpha}^{\beta}(x)$	Legendre functions	4

Symbol	Name of the Function	Listed under
$Q_\alpha^\beta(z)$, $Q_\alpha^\beta(x)$	Legendre functions	4
$s_{\alpha,\beta}(z)$, $S_{\alpha,\beta}(z)$	Lommel's function	8
$T_n(x)$, $U_n(x)$	Chebychev's polynomials	2
$W_{\alpha,\beta}(z)$	Whittaker's function	11
$Y_\nu(z)$	Neumann's function	5
$B(x,y)$	Beta function	3
$\Gamma(z)$	Gamma function	3